UNIT 7A – Geometric Figures

Angles Lesson 7Ae1 (7.G.5)

Classifying Angles:

- **Acute Angle**: less than 90°
- **Right Angle**: exactly 90°
- **Obtuse Angle**: between 90° and 180°
- **Straight Angle**: exactly 180°

Classify each angle as acute, obtuse, right, or straight.

1. $\angle LAT$ right
2. $\angle YKS$ acute
3. $\angle LAB$ straight
4. $\angle RAB$ obtuse
Naming Angles:

1. Use the vertex as the middle letter and a point form each side. $\angle LAR$

2. Use the vertex only $\angle K$

3. Use a number $\angle 2$

Name each Angle in 3 ways.

1. $\angle 1KS$
   
2. $\angle SKY$
   
3. $\angle LAR$

4. $\angle RAT$

5. $\angle TAN$

6. $\angle NAB$
**Vertical Angles**

**Vertical Angles:** Two angles are vertical if they are opposite angles formed by the intersection of two lines. Vertical angles are congruent (have the same measure).

Lines \(a\) and \(b\) intersect. \(\angle 1\) and \(\angle 3\) are vertical angles. Another pair is \(\angle 2\) and \(\angle 4\).

**Adjacent Angles:** Two angles are adjacent if they share a common vertex, a common side and do not overlap.

Refer to the diagram at the right. Identify each angle pair as *adjacent*, *vertical*, or *neither*.

7. \(\angle 1\) and \(\angle 2\) **adjacent**

8. \(\angle 2\) and \(\angle 5\) **neither**

9. \(\angle 1\) and \(\angle 3\) **vertical**

10. \(\angle 3\) and \(\angle 4\) **adjacent**

11. \(\angle 3\) and \(\angle 5\) **neither**

12. \(\angle 1\) and \(\angle 4\) **neither**
Finding a missing Measure

For the diagram above, if $m<2 = 95^0$ find $m<4$

** Since angle 2 and angle 4 are vertical angles they are congruent, therefore $m<4 = 95^0$

Use the figure at the right to answer the following questions

1. If $m<1 = 75^0$, what is $m<3$?

Use the figure at the right to answer the following questions

2. Find the value of $x$ in the figure?

$$2x = 120$$
$$\frac{2x}{2} = \frac{120}{2}$$
$$x = 60$$
Use the figure at the right to answer the following questions

3. Find the value of $x$ in the figure?

$$\frac{5x}{5} = \frac{55}{5}$$

$x = 11$

4. Find the value of $x$ in the figure?

$$3x - 4 + 4 = 146 + 4$$

$$3x = 150$$

$$x = 50$$

5. Find the value of $x$ in the figure?

$$2x + 6 = 80 - 6$$

$$2x = 74$$

$$x = 37$$
Complementary and Supplementary Angles Lesson 7A.2 (7.G.5)

**Complementary Angles**: Two angles whose sum of their measure is $90^\circ$

Angles $x$ and $y$ are adjacent because they share a ray (line in black) and a vertex (point in black called D).

Now why are they complementary?

Notice also that the angle in blue measures $90$ degrees

Since the measure of angle $x$ plus the measure of angle $y = 90$ degrees ($x + y = 90$ degrees), $x$ and $y$ are complementary.

Angles do not have to be adjacent to be complementary. The following angles are also complementary as long as the sum of the measures equal $90$ degrees.

- $50^\circ$
- $40^\circ$
- $20^\circ$
- $70^\circ$
**Supplementary Angles:** two angles whose sum of their measure is $180^\circ$

Again, angles $a$ and $b$ adjacent because they share a ray (line in black) and a vertex (point in black called D), so they are adjacent angles.

Now why are they supplementary?

Notice also that the angle in blue measures 180 degrees because the angle is a straight line and a straight line measures 180 degrees.

Since the measure of angle $a$ plus the measure of angle $b = 180$ degrees ($a + b = 90$ degrees), $a$ and $b$ are supplementary.

Again, angles do not have to be adjacent to be supplementary. The following angles are also supplementary as long as the sum of the measures equal 180 degrees.
A mnemonic to help you remember:
The C in Complementary stands for Corner, 90°
The S in Supplementary stands for Straight, 180°

Identify each pair of angles as complementary, supplementary, or neither

\[ 30° + 150° = 180° \]
The angles are supplementary.

\[ 16° + 74° = 90° \]
The angles are complementary.

Finding a missing Measure

Example:
\(x\) and \(y\) are supplementary angles. Given \(x = 72°\), find the value \(y\).

Solution:
\[ x + y = 180° \]
\[ 72° + y = 180° \]
\[ y = 180° - 72° = 108° \]

1. Find the missing angle measure.

In the diagram at the right,<br>\(<ABC\) and \(<CBD\) are supplementary. Find \(m <ABC\).
\[ \angle ABC = x \quad x + 30° = 180° \]
\[ \angle CBD = 30° - 30° - 30° \]
\[ x = 150° \]
\[ m <ABC = 150° \]
In the diagram at the right, \(<XYW\) and \(<VYW\) are complementary. Find \(m<XYW\)

\[
\begin{align*}
\angle XYW + 52 &= 90 \\
-52 &= -52 \\
\overline{m<XYW} &= 38^\circ
\end{align*}
\]

2. Use the figure at the right to answer the following questions

a. If \(<1 = 75^0\), what is \(m<2?\)
\[
\begin{align*}
75^0 + m<2 &= 180^\circ \\
-75 &= -75 \\
\overline{m<2} &= 105^\circ
\end{align*}
\]

b. Find \(m<3\)

\(m<3 = 75^0\) (vertical to \(<1\))

\(m<4 = 105^0\) (vertical to \(<2\))

3. Use the information given in the diagram to find \(x\).

\[
\begin{align*}
x + 44 &= 180 \\
-44 &= -44 \\
\overline{x} &= 136
\end{align*}
\]
Since the two angles form a straight line, they are supplementary. The sum of their measures is 180°.

\[ 5x + 35 = 180 \]

\[
\begin{array}{c c c}
-35 \\
\hline
5x \\
\hline
5
\end{array}
\]

\[ 5x = 145 \]

\[ \frac{5x}{5} = \frac{145}{5} \]

\[ x = 29 \]

Write the equation.

Subtract 35 from each side.

Simplify.

Divide each side by 5.

**Find the value of x.**

\[ 2x + 3x = 90 \]

\[ \frac{5x}{5} = \frac{90}{5} \]

\[ x = 18 \]

\[ x + x + 20 = 180 \]

\[ 2x + 20 = 180 \]

\[ 2x = 160 \]

\[ \frac{2x}{2} = \frac{160}{2} \]

\[ x = 80 \]
Identify each pair of angles as \textit{complementary}, \textit{supplementary}, or \textit{neither}.

1. \hspace{1cm} \text{Supplementary}

2. \hspace{1cm} \text{Complementary}

3. \hspace{1cm} \text{Neither}

**ALGEBRA** Find the value of $x$ in each figure.

4. \hspace{1cm} \frac{36}{6} = \frac{6x}{6}

\[ 6 = x \]

5. \hspace{1cm} 4x + 56 = 180

\[ -56 \hspace{1cm} -56 \]

\[ 4x = 124 \]

\[ \frac{x}{4} = \frac{124}{4} \]

\[ x = 31 \]

6. \hspace{1cm} 2x + 22 = 90

\[ -22 \hspace{1cm} -22 \]

\[ 2x = 68 \]

\[ \frac{x}{2} = \frac{68}{2} \]

\[ x = 34 \]
Triangles Lesson 7A

Triangles can be classified into three categories by their side lengths...

**Scalene Triangle** – all sides have different lengths

**Isosceles Triangle** – two sides have the same length

**Equilateral Triangle** – all sides have the same length

Triangles are classified into four categories by their angles.

**Acute Triangle** – all three angles are acute

**Equiangular Triangle** – all three angles are the same measure

**Obtuse Triangle** – one of the angles is obtuse

**Right Triangle** – one of the angles measures 90°
Classify each triangle by their side lengths and angle measures.

1. scalene
   obtuse

2. equilateral
   equiangular

3. scalene
   acute

4. isosceles
   acute
Finding missing angle measures

The sum of the interior angles of a triangle equals $180^\circ$.

Find the missing angle measures:

\[ 95 + 35 + x = 180 \]
\[ 130 + x = 180 \]
\[ -130 \]
\[ x = 50^\circ \]

\[ 52 + 90 + x = 180 \]
\[ 142 + x = 180 \]
\[ -142 \]
\[ x = 38^\circ \]

All sides same means all angles are the same, $60^\circ, 60^\circ, 60^\circ$.

\[ 3x + x + 2x = 180 \]
\[ 6x = 180 \]
\[ \frac{6x}{6} = \frac{180}{6} \]
\[ x = 30^\circ \]
\[ 2x = 60^\circ \]
\[ 3x = 90^\circ \]
Classify each triangle by its angles and by its sides.

1. \( \begin{align*}
&\text{1. acute} \\
&\text{2. scalene}
\end{align*} \)

2. \( \begin{align*}
&\text{1. right} \\
&\text{2. scalene}
\end{align*} \)

3. \( \begin{align*}
&\text{1. equiangular} \\
&\text{2. equilateral}
\end{align*} \)

4. \( \begin{align*}
&\text{1. right} \\
&\text{2. scalene}
\end{align*} \)

Find the value of \( x \).

5. \( \begin{align*}
&x + 90 + 40 = 180 \\
&\phantom{=} \quad x + 130 = 180 \\
&\quad \quad \quad -130 -130
\end{align*} \)

\( x = 50° \)

6. \( \begin{align*}
&x + 20 + 25 = 180 \\
&\phantom{=} \quad x + 45 = 180 \\
&\quad \quad \quad -45 -45
\end{align*} \)

\( x = 135° \)

7. \( \begin{align*}
&x + 30 + 130 = 180 \\
&\phantom{=} \quad x + 160 = 180 \\
&\quad \quad \quad -160 -160
\end{align*} \)

\( x = 20° \)

8. \( \begin{align*}
&x + 10 + 95 = 180 \\
&\phantom{=} \quad x + 135 = 180 \\
&\quad \quad \quad -135 -135
\end{align*} \)

\( x = 45° \)

9. \( \begin{align*}
&x + 1 + x + 2 + x = 180 \\
&3x + 3 = 180 \\
&\phantom{=} \quad -3 \quad -3 \\
&3x = 177 \\
&\phantom{=} \quad \frac{3}{3} \quad \frac{3}{3}
\end{align*} \)

\( x = 59° \)

\( x + 1 = 60° \\
\phantom{=} \quad x + 2 = 61° \)
**Scale Drawings Lesson 7A**

A *scale drawing* is an enlarged or reduced drawing of an object that is similar to an actual object. Maps and floor plans are smaller than the actual size. A scale drawing of a human cell is larger than the actual size.

***On a map, the equal sign in 1 in. = 30 mi does not mean that the two quantities are equal, as it would in an equation. A *scale* is the ratio that compares a length in a drawing to the corresponding length in the actual object. If a 30-mile road is 1 in. long on a map, you can write the scale of the map in these three ways:

\[
\frac{1\text{ in.}}{50\text{ mi}} = \frac{1\text{ in.}}{30\text{ mi}} = \frac{1\text{ in.}}{50\text{ mi}}
\]

**Using a Scale Drawing**

1. You have a scale drawing of a boat. The length of the boat on the drawing is 3 cm. What is the actual length of the boat?

Write the scale of the drawing, 1 cm = 1.5 m as \(\frac{1}{1.5}\). Then write a proportion in which each ratio compares centimeters to meters.

Let \(n\) represent the actual length of the boat.

\[
\frac{\text{drawing (cm)}}{\text{actual (m)}} = \frac{3}{1.5} = \frac{n}{4.5}
\]

\[n = 4.5\]  \(\text{Simplify.}\)

The actual length of the boat is 4.5 m.
2. Find the actual distance from Charlotte to Winston-Salem.

a) Use a centimeter ruler to find the map distance from Charlotte to Winston-Salem. The map distance is about 1.6 cm.

b) Use a proportion to find the actual distance. Let \( n \) represent the actual distance.

\[
\frac{1 \text{ cm}}{75 \text{ km}} = \frac{1.6 \text{ cm}}{n \text{ km}}
\]

\( n = \frac{1.6 \times 75}{75} = 1.2 \text{ km} \)

3. A scale model of a mosquito has a scale of 1 inch = 0.125 inch. If the length of the mosquito on the model is 3 inches, what is the actual length of the mosquito?

\[
\frac{1 \text{ inch}}{0.125 \text{ actual}} = \frac{3 \text{ inches}}{n}
\]

\( n = \frac{3}{0.125} = 24 \text{ inches} \)
4. On a map, the distance between City A and City B is 14 inches. If the scale on the map is 2 inches = 5 miles, what is the actual distance between City A and City B?

\[ \frac{2\text{ in}}{5\text{ mi}} = \frac{14\text{ in}}{n\text{ mi}} \]

\[ 2n = 14(5) \]

\[ 2n = 70 \]

\[ n = 35 \]

35 miles

5. Sylvia measured a house and its lot and made a scale drawing. She used the scale 1 inch = 9 feet. The backyard deck is 54 feet long in real life. How long is the deck in the drawing?

\[ \frac{1\text{ in}}{9\text{ ft}} = \frac{n}{54\text{ ft}} \]

\[ 1(54) = 9n \]

\[ 54 = 9n \]

\[ \frac{54}{9} = n \]

\[ 6 = n \]

6 inches

6. A particular motorcycle is 9 ft long. A model of it was built with a scale of 1 in : 3 ft. How long is the model?

\[ \frac{1\text{ in}}{3\text{ ft}} = \frac{n}{9\text{ ft}} \]

\[ 1(9) = 3n \]

\[ 9 = 3n \]

\[ \frac{9}{3} = n \]

\[ 3 = n \]

3 inches
7. A model train is 7 in tall. If it was built with a scale of 1 in : 2 ft then how tall is the real train?

\[ \frac{\text{in}}{2} = \frac{7}{n} \text{ ft.} \]

\[ 1n = \frac{7 \times 2}{1} \]

\[ 1n = 14 \text{ inches} \]

\[ 14 \text{ feet} \]

8. Find the distance between Riverside and Victoria if they are 9 cm apart on a map with a scale of 1 cm : 18 km.

\[ \frac{\text{cm}}{18} = \frac{9}{n} \text{ km} \]

\[ 1n = \frac{9 \times 18}{1} \]

\[ 1n = 162 \text{ km} \]

9. Milton and San Jose are 9 in apart on a map that has a scale of 1 in : 13 mi. How far apart are the real cities?

\[ \frac{\text{in}}{13} = \frac{9}{n} \text{ mi} \]

\[ 1n = \frac{9 \times 13}{1} \]

\[ 1n = 117 \text{ miles} \]
Finding the Scale Factor Lesson 7A.7h Continued (7.G.1)

A scale factor is a scale written as a ratio without units in simplest form.

Remember: ratios have the same unit of measure!!

Refer to the model boxcar shown above. The actual length of a boxcar is 609 in. What is the scale factor of the model?

\[
\frac{\text{scale length}}{\text{actual length}} = \frac{7}{609} = \frac{1}{87}
\]

The scale is 1 in. : 87 in.

1. Find the scale factor of a model sailboat if the scale is 1 inch = 6 feet.

   \[\text{1 in} : 72 \text{ in}\]

2. What is the scale factor of a model car if the scale is 1 inch = 2 feet?

   \[\text{1 in} : 24 \text{ in}\]

20
3. Rivertown and Marion are 108 miles from each other. If the distance on the map is 3 inches, find the scale of the map.

\[ 108 \div 3 = 36 \quad \text{1 in} = 36 \text{ miles} \]

4. Springfield and Chicago are approximately 202 miles apart. If the distance on the map is 2 inches, find the scale factor of the map.

\[ 202 \div 2 = 101 \quad \text{1 in} = 101 \text{ miles} \]

5. A model plane is 12 inches long. The actual plane is 40 feet. What is the scale factor of the model?

\[ \frac{12 \text{ in}}{480 \text{ in}} = \frac{1}{40} \]

6. The floor plans for a house measure the living room to be 3 inches by 5 inches. If the actual length of the living room is 18 feet, find the scale factor of the floor plans.

\[ 3 \text{ in} : 18 \text{ ft} \]

\[ \frac{18}{12} \]

\[ 5 \text{ in} : 216 \text{ in} \]
Draw Three-Dimensional Figures Lesson 7.5 (7.G.3)

SOLID - a three-dimensional figure

Example 1

Draw a top, a side, and a front view of the solid at the right.

The top view is a triangle. The side and front views are rectangles.

Example 2

Draw a top, a side, and a front view of the figure.

Example 3

Draw a top, a side, and a front view of the eraser shown.
Draw the top, a side, and a front view of the solid below.

Draw the top, a side, and a front view of the solid below.

Draw the top, a side, and a front view of each of the solid below.

1

2

3
Draw a corner view of the three-dimensional figure whose top, side, and front views are shown.

**Step 1** Use the top view to draw the base of the figure, a 1-by-3 rectangle.

**Step 2** Add edges to make the base a solid figure.

**Step 3** Use the side and front views to complete the figure.

Draw a corner view of the three-dimensional figure whose top, side, and front views are shown. Use isometric dot paper.
Draw a corner view of the three-dimensional figure whose top, side, and front views are shown. Use isometric dot paper.
Cross Sections Lesson 7A.6(7.G.3)

A polyhedron is a three-dimensional figure with flat surfaces that are polygons.

A prism is a polyhedron with two parallel, congruent faces called bases.

A pyramid is a polyhedron with one base that is a polygon and faces that are triangles.

<table>
<thead>
<tr>
<th>Prisms</th>
<th>Top and bottom bases are parallel</th>
<th>Shape of the base tells the name of the prism</th>
</tr>
</thead>
<tbody>
<tr>
<td>![Prism Diagram]</td>
<td>![Prism Diagram]</td>
<td>![Prism Diagram]</td>
</tr>
<tr>
<td>Cylinders</td>
<td>Top and bottom bases are parallel</td>
<td>No vertices and no edges</td>
</tr>
<tr>
<td>![Cylinder Diagram]</td>
<td>![Cylinder Diagram]</td>
<td>![Cylinder Diagram]</td>
</tr>
<tr>
<td>Pyramids</td>
<td>One base</td>
<td>Shape of the base tells the name of the pyramid</td>
</tr>
<tr>
<td>![Pyramid Diagram]</td>
<td>![Pyramid Diagram]</td>
<td>![Pyramid Diagram]</td>
</tr>
<tr>
<td>Cones</td>
<td>One base – always a circle</td>
<td>One vertex and no edges</td>
</tr>
<tr>
<td>![Cone Diagram]</td>
<td>![Cone Diagram]</td>
<td>![Cone Diagram]</td>
</tr>
</tbody>
</table>
Identify the figure. Then name the bases, faces, edges, and vertices.

The figure is a pentagonal prism.

The bases are \(ABCDE\) and \(FGHIJ\).

The faces are \(ABCDE, FGHIJ, ABGF, BCHG, CDIH, DEJI,\) and \(EAFJ\).

The edges are \(\overline{AB}, \overline{BC}, \overline{CD}, \overline{DE}, \overline{EA}, \overline{AF}, \overline{BG}, \overline{CH}, \overline{DI}, \overline{EJ}, \overline{FG}, \overline{CH}, \overline{HI}, \overline{IJ}, \overline{FF}\)

The vertices are \(A, B, C, D, E, F, G, H, I, J\).

Identify the figures. Then name the bases, faces, edges, and vertices.

The figure is a triangular prism

The bases are \(UVW\) and \(STR\)

The faces are \(SVWT, TRUW, RSVU\)

The edges are \(\overline{SV}, \overline{ST}, \overline{SR}, \overline{RT}, \overline{RU}, \overline{TW}, \overline{UV}, \overline{VW}, \overline{UV}\)

The vertices are \(S, T, R, U, V, W\)

rectangular pyramid

Base: \(IJHG\)

Faces: \(FIH, FHG, FGI, FJH\)

Edges: \(\overline{FI}, \overline{FH}, \overline{FG}, \overline{FI}, \overline{IJ}, \overline{JG}, \overline{GH}, \overline{HI}\)

Vertices: \(I, J, H, G, F\)
Cross Section: The intersection of a solid and a plane.

1. See the below diagram and then find shape resulting from the cross section.
2. Describe the shape resulting from the cross section.

3. Describe the shape resulting from the cross section.

4. Describe the shape resulting from the cross section.

5. Describe the shape resulting from the cross section.
6. Describe the shape resulting from the cross section.

7. Describe the shape resulting from the cross section.

8. Describe the shape resulting from the cross section.

9. Describe the shape resulting from the cross section.