Chapter 1
Ratios and Proportional Reasoning

Essential Question
HOW can you show that two objects are proportional?

Common Core State Standards
Content Standards
7.RP.1, 7.RP.2, 7.RP.2a, 7.RP.2b, 7.RP.2c, 7.RP.2d, 7.RP.3, 7.NS.3

Mathematical Practices
1, 2, 3, 4, 5, 6

Math in the Real World
Airplanes used for commercial flights travel at a speed of about 550 miles per hour.
Suppose an airplane travels 265 miles in one-half hour. Draw an arrow on the speedometer below to represent the speed of the airplane in miles per hour.

FOLDABLES® Study Organizer
1 Cut out the Foldable on page FL3 of this book.
2 Place your Foldable on page 92.
3 Use the Foldable throughout this chapter as you learn about proportional reasoning.
Review Vocabulary

**Functions**: A function is a relationship that assigns exactly one output value for each input value. The function rule is the operation performed on the input. Perform each indicated operation on the input 10. Then write each output in the organizer.

<table>
<thead>
<tr>
<th>Input</th>
<th>Rule</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>Add 2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Subtract 3</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Multiply by 4</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Divide by 5</td>
<td></td>
</tr>
</tbody>
</table>
What Do You Already Know?

Read each statement. Decide whether you agree (A) or disagree (D). Place a checkmark in the appropriate column and then justify your reasoning.

<table>
<thead>
<tr>
<th>Statement</th>
<th>A</th>
<th>D</th>
<th>Why?</th>
</tr>
</thead>
<tbody>
<tr>
<td>A rate is a ratio that compares two quantities with different units.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A relationship between two quantities is proportional.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>The cross products of the proportion ( \frac{a}{b} = \frac{c}{d} ) are ( ac ) and ( bd ).</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A linear relationship has a constant rate of change.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Slope can be expressed as ( \frac{\text{rise}}{\text{run}} ).</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>The graph of a direct variation always passes through the origin.</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

When Will You Use This?

Here are a few examples of how rates are used in the real world.

**Activity 1** Professional race car drivers race in qualifying runs to compete for a good starting position. Do you think that they could predict their actual race time based on qualifying times? Explain your reasoning.

**Activity 2** Go online at connectED.mcgraw-hill.com to read the graphic novel *The Go-Kart Race*. How long did it take Seth to complete 12 laps? How long is each lap?
Example 1
Write the ratio of wins to losses as a fraction in simplest form.

\[
\text{wins} \rightarrow \frac{10}{12} = \frac{5}{6}
\]

The ratio of wins to losses is \(\frac{5}{6}\).

Example 2
Determine whether the ratios 250 miles in 4 hours and 500 miles in 8 hours are equivalent.

Compare the ratios by writing them in simplest form.

250 miles : 4 hours = \(\frac{250}{4}\) or \(\frac{125}{2}\)

500 miles : 8 hours = \(\frac{500}{8}\) or \(\frac{125}{2}\)

The ratios are equivalent because they simplify to the same fraction.

Quick Check

Ratios  Write each ratio as a fraction in simplest form.

1. adults : students
2. students : buses
3. buses : people

Equivalent Ratios  Determine whether the ratios are equivalent. Explain.

4. 20 nails for every 5 shingles
   12 nails for every 3 shingles

5. 12 out of 20 doctors agree
   15 out of 30 doctors agree
**Inquiry Lab**

**Unit Rates**

**How can you use a bar diagram to solve a real-world problem involving ratios?**

When Jeremy gets his allowance, he agrees to save part of it. His savings and expenses are in the ratio 7:5. If his daily allowance is $3, find how much he saves each day.

**Hands-On Activity**

You can use a bar diagram to represent the ratio 7:5.

**Step 1**

Complete the bar diagram below by writing **savings**, **expenses**, and $3 in the correct boxes.

```
  |   |   |   |   |
  |   |   |   |   |
  |   |   |   |   |
  |   |   |   |   |
  |   |   |   |   |
  |   |   |   |   |
  |   |   |   |   |
  |   |   |   |   |
  |   |   |   |   |
  |   |   |   |   |
  |   |   |   |   |
  |   |   |   |   |
  |   |   |   |   |
```

Total amount = $3

(Daily Allowance)

**Step 2**

Let \( x \) represent each part of a bar. Write and solve an equation to find the amount of money each bar represents.

\[
7x + \frac{12}{12}x = 3
\]

Write the equation.

\[
12x = 3
\]

There are 12 parts in all.

\[
\frac{12x}{12} = \frac{3}{12}
\]

Division Property of Equality

\[
x = \frac{1}{12} 	ext{ or } 0.25
\]

Simplify.

**Step 3**

Determine the amount Jeremy saves each day. Since each part of the bar represents $0.25, Jeremy’s savings are represented by

\[
7 \times $0.25 = \$1.75
\]

So, Jeremy saves $0.25 each day.
Investigate

Work with a partner to answer the following question.

1. The ratio of the number of boys to the number of girls on the swim team is 4:2. If there are 24 athletes on the swim team, how many more boys than girls are there? Use a bar diagram to solve.

   [Diagram]
   Total athletes = \_\_\_\_

Analyze and Reflect

Work with a partner to answer the following question.

2. **Reason Inductively** Suppose the swim team has 24 athletes, but the ratio of boys to girls on the swim team is 3:5. How would the bar diagram change?

   [Write answer]

Create

3. **Model with Mathematics** Write a real-world problem that could be represented by the bar diagram shown below. Then solve your problem.

   [Diagram]
   Total amount = 220

4. **Inquiry** HOW can you use a bar diagram to solve a real-world problem involving ratios?

   [Write answer]
Pulse Rate  You can take a person's pulse by placing your middle and index finger on the underside of their wrist. Choose a partner and take their pulse for two minutes.

1. Record the results in the diagram below.

   \[ \text{beats} \hspace{1cm} \text{minutes} \]

2. Use the results from Exercise 1 to complete the bar diagram and determine the number of beats per minute for your partner.

\[
\begin{array}{c|c}
\text{Beats in 2 minutes} & \text{Number of beats in 1 minute.} \\
\hline
\text{Number of beats in 1 minute.} & \text{Number of beats in 1 minute.} \\
\end{array}
\]

So, your partner's heart beats \_\_\_ times per minute.

3. Use the results from Exercise 1 to determine the number of beats for \( \frac{1}{2} \) minute for your partner.

---

**Which Mathematical Practices did you use?**

Shade the circle(s) that applies.

1. Persevere with Problems
2. Reason Abstractly
3. Construct an Argument
4. Model with Mathematics
5. Use Math Tools
6. Attend to Precision
7. Make Use of Structure
8. Use Repeated Reasoning
Find a Unit Rate

A ratio that compares two quantities with different kinds of units is called a rate. When you found each other's pulse, you were actually finding the heart rate.

\[
\frac{160 \text{ beats}}{2 \text{ minutes}}
\]

The units beats and minutes are different.

When a rate is simplified so that it has a denominator of 1 unit, it is called a unit rate.

\[
\frac{80 \text{ beats}}{1 \text{ minute}}
\]

The denominator is 1 unit.

The table below shows some common unit rates.

<table>
<thead>
<tr>
<th>Rate</th>
<th>Unit Rate</th>
<th>Abbreviation</th>
<th>Name</th>
</tr>
</thead>
<tbody>
<tr>
<td>number of miles</td>
<td>miles per hour</td>
<td>mi/h or mph</td>
<td>average speed</td>
</tr>
<tr>
<td>1 hour</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>number of miles</td>
<td>miles per gallon</td>
<td>ml/gal or mpg</td>
<td>gas mileage</td>
</tr>
<tr>
<td>1 gallon</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>number of dollars</td>
<td>price per pound</td>
<td>dollars/lb</td>
<td>unit price</td>
</tr>
<tr>
<td>1 pound</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Example

1. Adrienne biked 24 miles in 4 hours. If she biked at a constant speed, how many miles did she ride in one hour?

\[
\frac{24 \text{ miles}}{4 \text{ hours}} = \frac{24 \text{ mi}}{4 \text{ h}}
\]

Write the rate as a fraction.

\[
= \frac{24}{4} \div \frac{4}{4}
\]

Divide the numerator and the denominator by 4.

\[
= 6 \text{ mi} \div 1 \text{ h}
\]

Simplify.

Adrienne biked 6 miles in one hour.

Got it? Do these problems to find out.

Find each unit rate. Round to the nearest hundredth if necessary.

a. $300 for 6 hours
b. 220 miles on 8 gallons
### Example

2. Find the unit price if it costs $2 for eight juice boxes.

$2 \text{ for eight boxes} = \frac{\$2}{8 \text{ boxes}}$  

\[
= \frac{\$2 \div 8}{8 \text{ boxes} \div 8}  
\]

\[
= \frac{\$0.25}{1 \text{ box}} 
\]

The unit price is $0.25 per juice box.

### Got it? Do this problem to find out.

**c.** Find the unit price if a 4-pack of mixed fruit sells for $2.12.

### Example

3. The prices of 3 different bags of dog food are given in the table. Which size bag has the lowest price per pound rounded to the nearest cent?

<table>
<thead>
<tr>
<th>Bag Size (lb)</th>
<th>Price ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>49.00</td>
</tr>
<tr>
<td>20</td>
<td>23.44</td>
</tr>
<tr>
<td>8</td>
<td>9.88</td>
</tr>
</tbody>
</table>

- 40-pound bag  
  \$49.00 \div 40 \text{ pounds} \approx \$1.23 \text{ per pound}

- 20-pound bag  
  \$23.44 \div 20 \text{ pounds} \approx \$1.17 \text{ per pound}

- 8-pound bag  
  \$9.88 \div 8 \text{ pounds} \approx \$1.24 \text{ per pound}

The 20-pound bag sells for the lowest price per pound.

### Got it? Do this problem to find out.

**d.** Tito wants to buy some peanut butter to donate to the local food pantry. Tito wants to buy as much peanut butter as possible. Which brand should he buy?

### Alternative Method

One 40-lb bag is equivalent to two 20-lb bags or five 8-lb bags. The cost for one 40-lb bag is $49, the cost for two 20-lb bags is about 2 \times \$23 or \$46, and the cost for five 8-lb bags is about 5 \times \$10 or \$50. So, the 20-lb bag has the lowest price per pound.
Example

4. Lexi painted 2 faces in 8 minutes at the Crafts Fair. At this rate, how many faces can she paint in 40 minutes?

Method 1 Draw a Bar Diagram

<table>
<thead>
<tr>
<th>time to paint one face</th>
<th>time to paint one face</th>
</tr>
</thead>
<tbody>
<tr>
<td>8 min</td>
<td>4 min</td>
</tr>
</tbody>
</table>

It takes 4 minutes to paint one face. In 40 minutes, Lexi can paint $40 \div 4$ or 10 faces.

Method 2 Find a Unit Rate

$$\text{2 faces in 8 minutes} = \frac{2 \text{ faces}}{8 \text{ min}} = \frac{0.25 \text{ face}}{1 \text{ min}}$$

Multiply the unit rate by 40 minutes.

$$\frac{0.25 \text{ face}}{1 \text{ min}} \cdot 40 \text{ min} = 10 \text{ faces}$$

Using either method, Lexi can paint 10 faces in 40 minutes.

Guided Practice

1. CD Express offers 4 CDs for $60. Music Place offers 6 CDs for $75. Which store offers the better buy? (Examples 1–3)

2. After 3.5 hours, Pasha had traveled 217 miles. If she travels at a constant speed, how far will she have traveled after 4 hours? (Example 4)

3. Write 5 pounds for $2.49 as a unit rate. Round to the nearest hundredth. (Example 2)

4. Building on the Essential Question Use an example to describe how a rate is a measure of one quantity per unit of another quantity.

Rate Yourself!

Are you ready to move on? Shade the section that applies.

YES  ?  NO

For more help, go online to access a Personal Tutor.
Find each unit rate. Round to the nearest hundredth if necessary.

(Examples 1 and 2)

1. 360 miles in 6 hours

3. 45.5 meters in 13 seconds

4. $7.40 for 5 pounds

5. Estimate the unit rate if 12 pairs of socks sell for $5.79. (Examples 1 and 2)

6. **MP Justify Conclusions** The results of a swim meet are shown. Who swam the fastest? Explain your reasoning. (Example 3)

<table>
<thead>
<tr>
<th>Name</th>
<th>Event</th>
<th>Time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tawni</td>
<td>50-m Freestyle</td>
<td>40.8</td>
</tr>
<tr>
<td>Pepita</td>
<td>100-m Butterfly</td>
<td>60.2</td>
</tr>
<tr>
<td>Susana</td>
<td>200-m Medley</td>
<td>112.4</td>
</tr>
</tbody>
</table>

7. Ben can type 153 words in 3 minutes. At this rate, how many words can he type in 10 minutes? (Example 4)

8. Kenji buys 3 yards of fabric for $7.47. Then he realizes that he needs 2 more yards. How much will the extra fabric cost? (Example 4)

9. The record for the Boston Marathon’s wheelchair division is 1 hour, 18 minutes, and 27 seconds.
   a. The Boston Marathon is 26.2 miles long. What was the average speed of the record winner of the wheelchair division? Round to the nearest hundredth.

   b. At this rate, about how long would it take this competitor to complete a 30-mile race?
10. At Tire Depot, a pair of new tires sells for $216. The manager's special advertises the same tires selling at a rate of $380 for 4 tires. How much do you save per tire if you purchase the manager's special?

H.O.T. Problems  Higher Order Thinking

11. Use Math Tools  Find examples of grocery item prices in a newspaper, on television, or on the Internet. Compare unit prices of two different brands of the same item. Explain which item is the better buy.

12. Find the Error  Seth is trying to find the unit price for a package of blank compact discs on sale at 10 for $5.49. Find his mistake and correct it.

\[ 10 \div 5.49 = 1.82 \text{ each} \]

Persevere with Problems  Determine whether each statement is sometimes, always, or never true. Give an example or a counterexample.

13. A ratio is a rate.

14. A rate is a ratio.

15. Justify Conclusions  A 96-ounce container of orange juice costs $4.80. At what price should a 128-ounce container be sold in order for the unit rate for both containers to be the same? Explain your reasoning.
Extra Practice

Find each unit rate. Round to the nearest hundredth if necessary.

16. 150 people for 5 classes
    
    \[ \frac{150 \text{ people}}{5 \text{ classes}} = \frac{30 \text{ people}}{1 \text{ class}} \]
    
    30 people per class

17. 815 Calories in 4 servings
    
    \[ \frac{815 \text{ Calories}}{4 \text{ servings}} = \frac{203.75 \text{ Calories}}{1 \text{ serving}} \]
    
    203.75 Calories per serving

18. $1.12 for 8.2 ounces

19. 144 miles on 4.5 gallons

20. **Justify Conclusions** A grocery store sells a 6-pack of bottled water for $3.79, a 9-pack for $4.50, and a 12-pack for $6.89. Which package costs the least per bottle? Explain your reasoning.

21. **Justify Conclusions** Dalia earns $108.75 for working 15 hours as a holiday helper wrapping gifts. At this rate, how much money will she earn if she works 18 hours the next week? Explain.

22. **Use Math Tools** Use the graph that shows the average number of heartbeats for an active adult brown bear and a hibernating brown bear.

   a. What does the point (2, 120) represent on the graph?

   b. What does the ratio of the y-coordinate to the x-coordinate for each pair of points on the graph represent?

   c. Use the graph to find the bear's average heart rate when it is active and when it is hibernating.
23. The table shows the number of hours a group of friends worked doing various jobs and the amount each earned. Select the correct hourly rate to complete the table. Then place a checkmark in the row for the person who had the greatest hourly rate.

<table>
<thead>
<tr>
<th>Hours Worked</th>
<th>Amount Earned ($)</th>
<th>Earnings per hour ($)</th>
<th>Greatest Hourly Rate?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Caleb</td>
<td>5</td>
<td>34.25</td>
<td></td>
</tr>
<tr>
<td>Jeremy</td>
<td>7.5</td>
<td>65.25</td>
<td></td>
</tr>
<tr>
<td>Maria</td>
<td>4.25</td>
<td>34.00</td>
<td></td>
</tr>
<tr>
<td>Rosa</td>
<td>8</td>
<td>54.00</td>
<td></td>
</tr>
</tbody>
</table>

24. Mrs. Ross needs to buy dish soap. There are four different sized containers. Sort the brands from least to greatest unit price. Round each unit price to the nearest thousandth.

<table>
<thead>
<tr>
<th>Brand</th>
<th>Unit Price (per ounce)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Least</td>
<td></td>
</tr>
<tr>
<td>Greatest</td>
<td></td>
</tr>
</tbody>
</table>

Which brand is the best buy?  

### Dish Soap Prices

<table>
<thead>
<tr>
<th>Brand</th>
<th>Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lots of Suds</td>
<td>$0.98 for 8 ounces</td>
</tr>
<tr>
<td>Bright Wash</td>
<td>$1.29 for 12 ounces</td>
</tr>
<tr>
<td>Spotless Soap</td>
<td>$3.14 for 30 ounces</td>
</tr>
<tr>
<td>Lemon Bright</td>
<td>$3.50 for 32 ounces</td>
</tr>
</tbody>
</table>

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### Common Core Spiral Review

**Solve. Write in simplest form. 5.NF.4**

25. \( \frac{1}{2} \times \frac{4}{7} = \)

26. \( \frac{2}{3} \times \frac{1}{6} = \)

27. \( \frac{1}{4} \div \frac{3}{8} = \)
Real-World Link

**Speed Skating** Dana is skating laps to train for a speed skating competition. She can skate 1 lap in 40 seconds.

1. Write a ratio in simplest form comparing Dana’s time to her number of laps.
   
   Dana’s time (s) \[ \frac{\text{Number of Laps}}{} \]

2. Suppose Dana skates for 20 seconds. How many laps will she skate?

3. Write the ratio of Dana’s time from Exercise 2 to her number of laps.
   
   Dana’s time (s) \[ \frac{\text{Number of Laps}}{} \]

4. How could you simplify the ratio you wrote in Exercise 3?

Which **Mathematical Practices** did you use?
Shade the circle(s) that applies.

1. Persevere with Problems
2. Reason Abstractly
3. Construct an Argument
4. Model with Mathematics
5. Use Math Tools
6. Attend to Precision
7. Make Use of Structure
8. Use Repeated Reasoning
Simplify a Complex Fraction

Fractions like $\frac{20}{1\frac{1}{2}}$ are called complex fractions. Complex fractions are fractions with a numerator, denominator, or both that are also fractions. Complex fractions are simplified when both the numerator and denominator are integers.

Examples

1. Simplify $\frac{1}{2}$.

Recall that a fraction can also be written as a division problem.

$$\frac{1}{4} \div 2$$

Write the complex fraction as a division problem.

$$= \frac{1}{4} \times \frac{1}{2}$$

Multiply by the reciprocal of 2, which is $\frac{1}{2}$.

$$= \frac{1}{8}$$

Simplify.

So, $\frac{1}{2}$ is equal to $\frac{1}{8}$.

2. Simplify $\frac{1}{2}$.

Write the fraction as a division problem.

$$\frac{1}{2} = 1 + \frac{1}{2}$$

Write the complex fraction as a division problem.

$$= \frac{1}{1} \times \frac{2}{1}$$

Multiply by the reciprocal of $\frac{1}{2}$, which is $\frac{2}{1}$.

$$= \frac{2}{1}$$

Simplify.

So, $\frac{1}{2}$ is equal to 2.

Got it? Do these problems to find out.

a. $\frac{2}{3}$

b. $\frac{6}{1\frac{1}{2}}$

c. $\frac{2}{3} \div 7$

d. $\frac{4}{2}$
Find Unit Rates

When the fractions of a complex fractions represent different units, you can find the unit rate.

Examples

3. Josiah can jog $1\frac{1}{3}$ miles in $\frac{1}{4}$ hour. Find his average speed in miles per hour.

Write a rate that compares the number of miles to hours.

$$\frac{1\frac{1}{3}}{\frac{1}{4}} \text{ mi} = \frac{\frac{4}{3}}{\frac{1}{4}}$$

Write the complex fraction as a division problem.

$$= \frac{4}{3} \div \frac{1}{4}$$

Write the mixed number as an improper fraction.

$$= \frac{4}{3} \times \frac{4}{1}$$

Multiply by the reciprocal of $\frac{1}{4}$, which is $\frac{4}{1}$.

$$= \frac{16}{3}$$ or $5\frac{1}{3}$

Simplify.

So, Josiah jogs at an average speed of $5\frac{1}{3}$ miles per hour.

4. Tia is painting her house. She paints $34\frac{1}{2}$ square feet in $\frac{3}{4}$ hour.

At this rate, how many square feet can she paint each hour?

Write a ratio that compares the number of square feet to hours.

$$\frac{34\frac{1}{2}}{\frac{3}{4}} \text{ ft}^2 = \frac{\frac{69}{2}}{\frac{3}{4}}$$

Write the complex fraction as a division problem.

$$= \frac{69}{2} \div \frac{3}{4}$$

Write the mixed number as an improper fraction.

$$= \frac{69}{2} \times \frac{4}{3}$$

Multiply by the reciprocal of $\frac{3}{4}$, which is $\frac{4}{3}$.

$$= \frac{276}{6}$$ or 46

Simplify.

So, Tia can paint 46 square feet per hour.

Got it? Do these problems to find out.

e. Mr. Ito is spreading mulch in his yard. He spreads $4\frac{2}{3}$ square yards in 2 hours. How many square yards can he mulch per hour?

f. Aubrey can walk $4\frac{1}{2}$ miles in $1\frac{1}{2}$ hours. Find her average speed in miles per hour.
Example

5. On Javier's soccer team, about $33\frac{1}{3}\%$ of the players have scored a goal. Write $33\frac{1}{3}\%$ as a fraction in simplest form.

\[
33\frac{1}{3}\% = \frac{33\frac{1}{3}}{100} = \frac{33}{100} \div 100 = \frac{100}{3} \div 100 = \frac{100}{3} \times \frac{1}{100} = \frac{100}{100} = \frac{1}{3}
\]

Definition of percent
Write the complex fraction as a division problem
Write $33\frac{1}{3}\%$ as an improper fraction
Multiply by the reciprocal of 100, which is $\frac{1}{100}$
Simplify.

So, about $\frac{1}{3}$ of Javier's team has scored a goal.

Guided Practice

Simplify. (Examples 1 and 2)

1. \(\frac{18}{4} = \) 

2. \(\frac{3}{4} = \) 

3. \(\frac{1}{4} = \) 

4. Pep Club members are making spirit buttons. They make 490 spirit buttons in 3 1/2 hours. Find the number of buttons the Pep Club makes per hour. (Examples 3 and 4)

5. A county sales tax is $6\frac{2}{3}\%$. Write the percent as a fraction in simplest form. (Example 5)

6. Building on the Essential Question What is a complex fraction?

Rate Yourself!

How confident are you about simplifying complex fractions? Check the box that applies.

For more help, go online to access a Personal Tutor.
**Independent Practice**

**Simplify.** (Examples 1 and 2)

1. \( \frac{1}{2} \)  
2. \( \frac{2}{3} \)  
3. \( \frac{8}{9} \)  
4. \( \frac{2}{9} \)  
5. \( \frac{4}{5} \)  
6. \( \frac{1}{7} \)

7. Mary is making pillows for her Life Skills class. She bought \( 2 \frac{1}{2} \) yards of fabric. Her total cost was \$15. What was the cost per yard? (Examples 3 and 4)

8. Doug entered a canoe race. He rowed \( 3 \frac{1}{2} \) miles in \( \frac{1}{2} \) hour. What is his average speed in miles per hour? (Examples 3 and 4)

9. Monica reads \( 7 \frac{1}{2} \) pages of a mystery book in 9 minutes. What is her average reading rate in pages per minute? (Examples 3 and 4)

**Write each percent as a fraction in simplest form.** (Example 5)

10. \( 56\frac{1}{4}\% \)  
11. \( 15\frac{3}{5}\% \)  
12. \( 13\frac{1}{3}\% \)  

13. A bank is offering home loans at an interest rate of \( 5\frac{1}{2}\% \). Write the percent as a fraction in simplest form. (Example 5)
14. **Be Precise**  Kari measured the wingspan of the butterfly and the moth shown below. How many times larger is the moth than the butterfly?

![Black Swallowtail Butterfly](image1)  \[ \frac{3\frac{1}{2}}{7} \text{ in.} \]

![Hummingbird Moth](image2)  \[ \frac{3\frac{1}{2}}{7} \text{ in.} \]

15. **Construct an Argument**  Explain how complex fractions can be used to solve problems involving ratios.

16. **Reason Inductively**  Write three different complex fractions that simplify to \( \frac{1}{4} \).

17. **Persevere with Problems**  Use mental math to find the value of \( \frac{15}{124} \times \frac{230}{30} \div \frac{230}{124} \).

18. **Justify Conclusions**  The value of a mutual fund increased by \( 3\frac{1}{8}\% \). Write \( 3\frac{1}{8}\% \) as a fraction in simplest form. Justify your answer.

19. **Persevere with Problems**  The distance around the tire of a motorized scooter is 21.98 inches. The tires make one revolution every \( \frac{1}{10} \) second. Find the speed of the scooter in miles per hour. Round to the nearest tenth. (Hint: The speed of an object spinning in a circle is equal to the distance around the circle divided by the time it takes to complete one revolution.)

---

22  Chapter 1  Ratios and Proportional Reasoning
Extra Practice

Simplify.

20. \( \frac{1}{4} = \frac{4}{4} = 1 \div \frac{1}{4} \)

21. \( \frac{12}{3} = \) ______ 

22. \( \frac{9}{10} = \) ______ 

23. \( \frac{2}{1} = \) ______ 

24. \( \frac{12}{5} = \) ______ 

25. \( \frac{5}{6} = \) ______ 

26. Mrs. Frasier is making costumes for the school play. Each costume requires 0.75 yard of fabric. She bought 6 yards of fabric. How many costumes can Mrs. Frasier make?

27. A lawn company advertises that they can spread 7,500 square feet of grass seed in 2 1/2 hours. Find the number of square feet of grass seed that can be spread per hour.

Write each percent as a fraction in simplest form.

28. \( 2\frac{2}{5}\% = \) ______ 

29. \( 7\frac{3}{4}\% = \) ______ 

30. \( 8\frac{1}{3}\% = \) ______ 

31. **MP Justify Conclusions** The value of a certain stock increased by 1 1/4%.

Explain how to write 1 1/4% as a fraction in simplest form.
32. Debra bought \(3 \frac{1}{4}\) yards of fabric at a remnant sale for $13. Determine if each of the following remnant deals have the same unit price as Debra’s deal. Select yes or no.
   a. \(4\frac{2}{3}\) yards for $16  
      □ Yes  □ No
   b. \(2\frac{3}{4}\) yards for $11  
      □ Yes  □ No
   c. \(6\frac{1}{2}\) yards for $26  
      □ Yes  □ No

33. The table shows the distances traveled by 4 cyclists. Sort the speeds of the riders, in miles per hour, from slowest to fastest.

<table>
<thead>
<tr>
<th>Rider</th>
<th>Distance</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elena</td>
<td>(20\frac{1}{2}) mi</td>
<td>(2\frac{1}{2}) h</td>
</tr>
<tr>
<td>Julio</td>
<td>(12\frac{1}{2}) mi</td>
<td>(1\frac{1}{2}) h</td>
</tr>
<tr>
<td>Kevin</td>
<td>(20\frac{3}{4}) mi</td>
<td>(1\frac{3}{4}) h</td>
</tr>
<tr>
<td>Lorena</td>
<td>(33\frac{1}{2}) mi</td>
<td>(2\frac{1}{3}) h</td>
</tr>
</tbody>
</table>

Which rider had the fastest rate of speed? ________

---

**Common Core Spiral Review**

Fill in each box with the equivalent customary measurement. \(5.MD.1\)

34. 2 feet = □ inches
35. 5 tons = □ pounds
36. 8 gallons = □ quarts

Fill in each box with the equivalent metric measurement. \(5.MD.1\)

37. 1 meter = □ centimeters
38. 1 liter = □ milliliters
39. 1 kilogram = □ grams

---

24 Need more practice? Download more Extra Practice at connectED.mcgraw-hill.com.
Animals: Squirrels, chipmunks, and rabbits are capable of running at fast speeds. The table shows the top running speeds of these animals.

<table>
<thead>
<tr>
<th>Animal</th>
<th>Speed (mph)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Squirrel</td>
<td>10</td>
</tr>
<tr>
<td>Chipmunk</td>
<td>15</td>
</tr>
<tr>
<td>Cottonail Rabbit</td>
<td>30</td>
</tr>
</tbody>
</table>

1. How many feet are in 1 mile? 10 miles?
   1 mile = ______ feet
   10 miles = ______ feet

2. How many seconds are in 1 minute? 1 hour?
   1 minute = ______ seconds
   1 hour = ______ seconds

3. How could you determine the number of feet per second a squirrel can run?

4. Complete the following statement. Round to the nearest tenth. 10 miles per hour ≈ ______ feet per second

Which Mathematical Practices did you use?
Shade the circle(s) that applies.

① Persevere with Problems  ② Reason Abstractly  ③ Construct an Argument  ④ Model with Mathematics  ⑤ Use Math Tools  ⑥ Attend to Precision  ⑦ Make Use of Structure  ⑧ Use Repeated Reasoning
Convert Rates

The relationships among some commonly used customary and metric units of measure are shown in the tables below.

<table>
<thead>
<tr>
<th>Customary Units of Measure</th>
<th>Metric Units of Measure</th>
</tr>
</thead>
<tbody>
<tr>
<td>Smaller</td>
<td>Larger</td>
</tr>
<tr>
<td>12 inches</td>
<td>1 foot</td>
</tr>
<tr>
<td>16 ounces</td>
<td>1 pound</td>
</tr>
<tr>
<td>8 pints</td>
<td>1 gallon</td>
</tr>
<tr>
<td>3 feet</td>
<td>1 yard</td>
</tr>
<tr>
<td>5,280 feet</td>
<td>1 mile</td>
</tr>
</tbody>
</table>

Each of the relationships in the tables can be written as a unit ratio. Like a unit rate, a unit ratio is one in which the denominator is 1 unit. Below are three examples of unit ratios.

\[
\frac{12\text{ inches}}{1\text{ foot}} \quad \frac{16\text{ ounces}}{1\text{ pound}} \quad \frac{100\text{ centimeters}}{1\text{ meter}}
\]

The numerator and denominator of each of the unit ratios shown are equal. So, the value of each ratio is 1.

You can convert one rate to an equivalent rate by multiplying by a unit ratio or its reciprocal. When you convert rates, you include the units in your computation.

The process of including units of measure as factors when you compute is called dimensional analysis.

\[
\frac{10\text{ ft}}{1\text{ s}} = \frac{10 \times \frac{12\text{ in.}}{1\text{ ft}}}{1\text{ s}} = \frac{10 \times 12\text{ in.} \cdot 1}{1\text{ s} \cdot 1} = \frac{120\text{ in.}}{1\text{ s}}
\]

Example

1. A remote control car travels at a rate of 10 feet per second. How many inches per second is this?

\[
\frac{10\text{ ft}}{1\text{ s}} = \frac{10 \times \frac{12\text{ in.}}{1\text{ ft}}}{1\text{ s}} \quad \text{Use 1 foot = 12 inches. Multiply by } \frac{12\text{ in.}}{1\text{ ft}}.
\]

\[
= \frac{10 \times 12\text{ in.}}{1\text{ s}} \quad \text{Divide out common units.}
\]

\[
= \frac{10 \times 12\text{ in.}}{1\text{ s} \cdot 1} \quad \text{Simplify.}
\]

\[
= \frac{120\text{ in.}}{1\text{ s}} \quad \text{Simplify.}
\]

So, 10 feet per second equals 120 inches per second.
Examples

2. A swordfish can swim at a rate of 60 miles per hour. How many feet per hour is this?

You can use 1 mile = 5,280 feet to convert the rates.

\[
\frac{60 \text{ mi}}{1 \text{ h}} = \frac{60 \text{ mi}}{1 \text{ h}} \cdot \frac{5,280 \text{ ft}}{1 \text{ mi}}
\]

Multiply by \( \frac{5,280 \text{ ft}}{1 \text{ mi}} \).

\[
= \frac{60 \text{ mi}}{1 \text{ h}} \cdot \frac{5,280 \text{ ft}}{1 \text{ mi}}
\]

Divide out common units.

\[
= \frac{60 \cdot 5,280 \text{ ft}}{1 \cdot 1 \text{ h}}
\]

Simplify.

\[
= \frac{316,800 \text{ ft}}{1 \text{ h}}
\]

Simplify.

A swordfish can swim at a rate of 316,800 feet per hour.

3. Marvin walks at a speed of 7 feet per second. How many feet per hour is this?

You can use 60 seconds = 1 minute and you can use 60 minutes = 1 hour to convert the rates.

\[
\frac{7 \text{ ft}}{1 \text{ s}} = \frac{7 \text{ ft}}{1 \text{ s}} \cdot \frac{60 \text{ s}}{1 \text{ min}} \cdot \frac{60 \text{ min}}{1 \text{ h}}
\]

Multiply by \( \frac{60 \text{ s}}{1 \text{ min}} \) and \( \frac{60 \text{ min}}{1 \text{ h}} \).

\[
= \frac{7 \text{ ft}}{1 \text{ s}} \cdot \frac{60 \text{ s}}{1 \text{ min}} \cdot \frac{60 \text{ min}}{1 \text{ h}}
\]

Divide out common units.

\[
= \frac{7 \cdot 60 \cdot 60 \text{ ft}}{1 \cdot 1 \cdot 1 \text{ h}}
\]

Simplify.

\[
= \frac{25,200 \text{ ft}}{1 \text{ h}}
\]

Simplify.

Marvin walks 25,200 feet in 1 hour.

Got it? Do these problems to find out.

a. A gull can fly at a speed of 22 miles per hour. About how many feet per hour can the gull fly?

b. An AMTRAK train travels at 125 miles per hour. Convert the speed to miles per minute. Round to the nearest tenth.
Example

4. The average speed of one team in a relay race is about 10 miles per hour. What is this speed in feet per second?

We can use 1 mile = 5,280 feet, 1 hour = 60 minutes, and 1 minute = 60 seconds to convert the rates.

\[
\frac{10 \text{ mi}}{1 \text{ h}} \cdot \frac{5,280 \text{ ft}}{1 \text{ mi}} \cdot \frac{1 \text{ h}}{60 \text{ min}} \cdot \frac{1 \text{ min}}{60 \text{ s}}
\]

Multiply by distance and time unit ratios.

\[
= \frac{10 \text{ mi}}{1 \text{ h}} \cdot \frac{5,280 \text{ ft}}{1 \text{ mi}} \cdot \frac{1 \text{ h}}{60 \text{ min}} \cdot \frac{1 \text{ min}}{60 \text{ s}}
\]

Divide out common units.

\[
= \frac{10 \cdot 5,280}{1 \cdot 1 \cdot 60 \cdot 60}
\]

Simplify.

\[
= \frac{52,800}{3,600}
\]

Simplify.

\[
\approx \frac{14.7}{1}
\]

Simplify.

The relay team runs at an average speed of about 14.7 feet per second.

Guided Practice

1. Water weighs about 8.34 pounds per gallon. About how many ounces per gallon is the weight of the water? (Examples 1 and 2)

2. A skydiver is falling at about 176 feet per second. How many feet per minute is he falling? (Example 3)

3. Lorenzo rides his bike at a rate of 5 yards per second. About how many miles per hour can Lorenzo ride his bike? (Hint: 1 mile = 1,760 yards) (Example 4)

4. Building on the Essential Question: Explain why the ratio \(\frac{3 \text{ feet}}{1 \text{ yard}}\) has a value of one.

Rate Yourself!

- I understand how to convert unit rates. [ ]
  - Great! You're ready to move on!

- I still have questions about converting unit rates. [ ]
  - No Problem! Go online to access a Personal Tutor.
1. A go-kart's top speed is 607,200 feet per hour. What is the speed in miles per hour? (Examples 1 and 2)

2. The fastest a human has ever run is 27 miles per hour. How many miles per minute did the human run? (Example 3)

3. A peregrine falcon can fly 322 kilometers per hour. How many meters per hour can the falcon fly? (Example 3)

4. A pipe is leaking at 1.5 cups per day. About how many gallons per week is the pipe leaking? (Hint: 1 gallon = 16 cups) (Example 4)

5. Charlie runs at a speed of 3 yards per second. About how many miles per hour does Charlie run? (Example 4)

6. **Model with Mathematics** Refer to the graphic novel frame below. Seth traveled 1 mile in 57.1 seconds. About how does Seth travel in miles per hour?

---

**I can't believe how fast I was going.**
7. The speed at which a certain computer can access the Internet is 2 megabytes per second. How fast is this in megabytes per hour?

8. **Use Math Tools** The approximate metric measurement of length is given for a U.S. customary unit of length. Use your estimation skills to complete the graphic organizer below. Fill in each blank with foot, yard, inch, or mile.

<table>
<thead>
<tr>
<th>Metric</th>
<th>Customary</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.54 centimeters</td>
<td>1</td>
</tr>
<tr>
<td>0.30 meter</td>
<td>1</td>
</tr>
<tr>
<td>0.91 meter</td>
<td>1</td>
</tr>
<tr>
<td>1.61 kilometers</td>
<td>1</td>
</tr>
</tbody>
</table>

**H.O.T. Problems** Higher Order Thinking

9. **Which One Doesn’t Belong?** Circle the rate that does not belong with the other three. Explain your reasoning.

| 60 mi/h     | 88 ft/s    | 500 ft/min | 1,440 mi/day |

10. **Reason Inductively** When you convert 100 feet per second to inches per second, will there be more or less than 100 inches. Explain.

11. **Persevere with Problems** Use the information in Exercise 8 to convert 7 meters per minute to yards per hour. Round to the nearest tenth.

12. **Model with Mathematics** Write and solve a real-world problem in which a rate is converted.
13. 20 mi/h = \( \boxed{1,760} \) ft/min

\[
\frac{20 \text{ mi}}{1 \text{ h}} \cdot \frac{5,280 \text{ ft}}{1 \text{ mi}} \cdot \frac{1 \text{ h}}{60 \text{ min}} =
\]

\[
\frac{105,600 \text{ ft}}{60 \text{ min}} = 1,760 \text{ ft/min}
\]

14. 16 cm/min = \( \boxed{\text{m/h}} \)

\[
\frac{16 \text{ cm}}{1 \text{ min}} \cdot \frac{1 \text{ m}}{100 \text{ cm}} \cdot \frac{60 \text{ min}}{1 \text{ h}} =
\]

\[
\frac{960 \text{ m}}{100 \text{ h}} = 9.6 \text{ m/h}
\]

15. 45 mi/h = \( \boxed{\text{ft/s}} \)

16. 26 cm/s = \( \boxed{\text{m/min}} \)

17. 24 mi/h = \( \boxed{\text{ft/s}} \)

18. 105.6 L/h = \( \boxed{\text{L/min}} \)

19. The table shows the speed and number of wing beats per second for various flying insects.
   a. What is the speed of a housefly in feet per second? Round to the nearest hundredth.
   b. How many times does a dragonfly’s wing beat per minute?
   c. About how many miles can a bumblebee travel in one minute?
   d. How many times can a honeybee beat its wings in one hour?
20. A model airplane flew a distance of 330 feet in 15 seconds. Select all of the unit rates that are equivalent to the speed of the model airplane.
  - 15 miles per hour
  - 12 miles per hour
  - 1320 feet per minute
  - 1,056 feet per minute

21. The table shows how far some of the world’s fastest animals can run in different lengths of time at their top speed. Select the correct top speed to complete the table.

<table>
<thead>
<tr>
<th>Animal</th>
<th>Top Speed (mph)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cheetah</td>
<td>45, 60, 50, 65</td>
</tr>
<tr>
<td>Elk</td>
<td></td>
</tr>
<tr>
<td>Lion</td>
<td>55, 70</td>
</tr>
<tr>
<td>Quarter Horse</td>
<td></td>
</tr>
</tbody>
</table>

Which animal had the fastest rate of speed?

---

**Common Core Spiral Review**

Determine if each pair of rates are equivalent. Explain your reasoning.

6.RP.3b

22. $36 for 4 baseball hats; $56 for 7 baseball hats

23. 12 posters for 36 students; 21 posters for 63 students

24. An employer pays $22 for 2 hours. Use the ratio table to determine how much she charges for 5 hours. 6.RP.3b

<table>
<thead>
<tr>
<th>Payment</th>
<th>$22</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hours</td>
<td>2</td>
</tr>
</tbody>
</table>

---

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**Pizza Party** Ms. Cochran is planning a year-end pizza party for her students. Ace Pizza offers free delivery and charges $8 per medium pizza.

1. Complete the table to determine the cost for different numbers of pizzas ordered.

<table>
<thead>
<tr>
<th>Cost ($)</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pizza</td>
<td>1 2 3 4 5</td>
</tr>
</tbody>
</table>

2. For each number of pizzas, fill in the boxes to write the relationship of the cost and number of pizzas as a ratio in simplest form.

\[
\frac{16}{2} = \quad \quad \quad \quad \frac{24}{3} = \quad \quad \quad \quad \frac{32}{5} =
\]

3. What do you notice about the simplified ratios?

**Which [MP] Mathematical Practices did you use?**

Shade the circle(s) that applies.

1. Persevere with Problems
2. Reason Abstractly
3. Construct an Argument
4. Model with Mathematics
5. Use Math Tools
6. Attend to Precision
7. Make Use of Structure
8. Use Repeated Reasoning
Identify Proportional Relationships

Two quantities are proportional if they have a constant ratio or unit rate. For relationships in which this ratio is not constant, the two quantities are nonproportional.

In the pizza example on the previous page, the cost of an order is proportional to the number of pizzas ordered.

\[
\frac{\text{cost of order}}{\text{pizzas ordered}} = \frac{8}{1} = \frac{16}{2} = \frac{24}{3} = \frac{32}{4} = \frac{40}{5} \text{ or } \$8 \text{ per pizza}
\]

All of the ratios above are equivalent ratios because they all have the same value.

Example

1. Andrew earns $18 per hour for mowing lawns. Is the amount of money he earns proportional to the number of hours he spends mowing? Explain.

Find the amount of money he earns for working a different number of hours. Make a table to show these amounts.

<table>
<thead>
<tr>
<th>Earnings ($)</th>
<th>18</th>
<th>36</th>
<th>54</th>
<th>72</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time (h)</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>

For each number of hours worked, write the relationship of the amount he earned and hour as a ratio in simplest form.

\[
\frac{\text{amount earned}}{\text{number of hours}} \rightarrow \frac{18}{1} \text{ or } 18 \quad \frac{36}{2} \text{ or } 18 \quad \frac{54}{3} \text{ or } 18 \quad \frac{72}{4} \text{ or } 18
\]

All of the ratios between the two quantities can be simplified to 18.

The amount of money he earns is proportional to the number of hours he spends mowing.

Got it? Do this problem to find out.

a. At Lakeview Middle School, there are 2 homeroom teachers assigned to every 48 students. Is the number of students at this school proportional to the number of teachers? Explain your reasoning.
**Examples**

2. Uptown Tickets charges $7 per baseball game ticket plus a $3 processing fee per order. Is the cost of an order proportional to the number of tickets ordered? Explain.

<table>
<thead>
<tr>
<th>Cost ($)</th>
<th>10</th>
<th>17</th>
<th>24</th>
<th>31</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tickets Ordered</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>

For each number of tickets, write the relationship of the cost and number of tickets as a ratio in simplest form.

\[
\frac{\text{cost of order}}{\text{tickets ordered}} \rightarrow \frac{10}{1} \text{ or } 10 \quad \frac{17}{2} \text{ or } 8.5 \quad \frac{24}{3} \text{ or } 8 \quad \frac{31}{4} \text{ or } 7.75
\]

Since the ratios of the two quantities are not the same, the cost of an order is *not* proportional to the number of tickets ordered.

3. You can use the recipe shown to make a fruit punch. Is the amount of sugar used proportional to the amount of mix used? Explain.

Find the amount of sugar and mix needed for different numbers of batches. Make a table to help you solve.

<table>
<thead>
<tr>
<th>Cups of Sugar</th>
<th>(\frac{1}{2})</th>
<th>1</th>
<th>(1\frac{1}{2})</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Envelopes of Mix</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>

For each number of cups of sugar, write the relationship of the cups and number of envelopes of mix as a ratio in simplest form.

\[
\frac{\text{cups of sugar}}{\text{envelopes of mix}} \rightarrow \frac{\frac{1}{2}}{1} \text{ or } 0.5 \quad \frac{1}{2} \text{ or } 0.5 \quad \frac{1\frac{1}{2}}{3} \text{ or } 0.5 \quad \frac{2}{4} \text{ or } 0.5
\]

All of the ratios between the two quantities can be simplified to 0.5. The amount of mix used is proportional to the amount of sugar used.

**Got it?** Do this problem to find out.

b. At the beginning of the year, Isabel had $120 in the bank. Each week, she deposits another $20. Is her account balance proportional to the number of weeks of deposits? Use the table below. Explain your reasoning.

<table>
<thead>
<tr>
<th>Time (wk)</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Balance ($)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Example

4. The tables shown represent the number of pages Martin and Gabriel read over time. Which situation represents a proportional relationship between the time spent reading and the number of pages read? Explain.

<table>
<thead>
<tr>
<th>Pages Martin Read</th>
<th>2</th>
<th>4</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time (min)</td>
<td>5</td>
<td>10</td>
<td>15</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Pages Gabriel Read</th>
<th>3</th>
<th>4</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time (min)</td>
<td>5</td>
<td>10</td>
<td>15</td>
</tr>
</tbody>
</table>

Write the ratios for each time period in simplest form.

$$\frac{\text{pages}}{\text{minutes}} = \frac{2}{5}, \frac{4}{10} \text{ or } \frac{2}{5}, \frac{6}{15} \text{ or } \frac{2}{5}, \frac{3}{5}, \frac{4}{10} \text{ or } \frac{3}{5}, \frac{7}{15}$$

All of the ratios between Martin’s quantities are \(\frac{2}{5}\). So, Martin’s reading rate represents a proportional relationship.

Guided Practice

1. The Vista Marina rents boats for $25 per hour. In addition to the rental fee, there is a $12 charge for fuel. Use a table to determine if the number of hours you rent the boat is proportional to the total cost. Explain. (Examples 1–3)

<table>
<thead>
<tr>
<th>Rental Time (h)</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost ($)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. Which situation represents a proportional relationship between the hours worked and amount earned for Matt and Jane? Explain. (Example 4)

<table>
<thead>
<tr>
<th>Matt’s Earnings ($)</th>
<th>12</th>
<th>20</th>
<th>31</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time (h)</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Jane’s Earnings ($)</th>
<th>12</th>
<th>24</th>
<th>36</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time (h)</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
</tbody>
</table>


Rate Yourself!

How confident are you about determining proportional relationships? Shade the ring on the target.

For more help, go online to access a Personal Tutor.
For Exercises 1 and 2, use a table to solve. Then explain your reasoning. (Examples 1 and 2)

1. An adult elephant drinks about 225 liters of water each day. Is the number of days the water supply lasts proportional to the number of liters of water the elephant drinks?

<table>
<thead>
<tr>
<th>Time (days)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water (L)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. An elevator ascends, or goes up, at a rate of 750 feet per minute. Is the height to which the elevator ascends proportional to the number of minutes it takes to get there? (Examples 1–3)

<table>
<thead>
<tr>
<th>Time (min)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Height (ft)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

3. Which situation represents a proportional relationship between the number of laps run by each student and their time? (Example 4)

<table>
<thead>
<tr>
<th>Desmond's Time (s)</th>
<th>146</th>
<th>292</th>
<th>584</th>
</tr>
</thead>
<tbody>
<tr>
<td>Laps</td>
<td>2</td>
<td>4</td>
<td>8</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Maria's Time (s)</th>
<th>150</th>
<th>320</th>
<th>580</th>
</tr>
</thead>
<tbody>
<tr>
<td>Laps</td>
<td>2</td>
<td>4</td>
<td>6</td>
</tr>
</tbody>
</table>

Copy and Solve Use a table to help you solve. Then explain your reasoning. Show your work on a separate piece of paper.

4. Plant A is 18 inches tall after one week, 36 inches tall after two weeks, 54 inches tall after three weeks. Plant B is 18 inches tall after one week, 36 inches tall after two weeks, 54 inches tall after three weeks. Which situation represents a proportional relationship between the plants' height and number of weeks? (Example 4)

5. Determine whether the measures for the figure shown are proportional.
   a. the length of a side and the perimeter
   b. the length of a side and the area
6. **Justify Conclusions**  MegaMart collects a sales tax equal to $\frac{1}{16}$ of the retail price of each purchase. The tax is sent to the state government.

a. Is the amount of tax collected proportional to the cost of an item before tax is added? Explain.

<table>
<thead>
<tr>
<th>Retail Price ($)</th>
<th>16</th>
<th>32</th>
<th>48</th>
<th>64</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tax Collected ($)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

b. Is the amount of tax collected proportional to the cost of an item after tax has been added? Explain.

<table>
<thead>
<tr>
<th>Retail Price ($)</th>
<th>16</th>
<th>32</th>
<th>48</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tax Collected ($)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cost Including Tax ($)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

---

**H.O.T. Problems**  Higher Order Thinking

7. **Find the Error**  Blake ran laps around the gym. His times are shown in the table. Blake is trying to decide whether the number of laps is proportional to the time. Find his mistake and correct it.

<table>
<thead>
<tr>
<th>Time (min)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Laps</td>
<td>4</td>
<td>6</td>
<td>8</td>
<td>10</td>
</tr>
</tbody>
</table>

It is proportional because the number of laps always increases by 2.

---

8. **Persevere with Problems**  Determine whether the cost for ordering multiple items that will be delivered is sometimes, always, or never proportional. Explain your reasoning.

---

9. **Model with Mathematics**  Give real-world examples of two similar situations in which one is a proportional relationship and the second one is nonproportional.

---

38  Chapter 1  Ratios and Proportional Reasoning
Extra Practice

For Exercises 10–12, use a table to solve. Then explain your reasoning.

10. A vine grows 7.5 feet every 5 days. Is the length of the vine on the last day proportional to the number of days of growth?

   Yes; the length to time ratios are all equal to 1.5 ft per day.

<table>
<thead>
<tr>
<th>Time (days)</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length (ft)</td>
<td>7.5</td>
<td>15</td>
<td>22.5</td>
<td>30</td>
</tr>
</tbody>
</table>

11. **STEM** To convert a temperature in degrees Celsius to degrees Fahrenheit, multiply the Celsius temperature by \(\frac{9}{5}\) and then add 32°. Is a temperature in degrees Celsius proportional to its equivalent temperature in degrees Fahrenheit?

<table>
<thead>
<tr>
<th>Degrees Celsius</th>
<th>0</th>
<th>10</th>
<th>20</th>
<th>30</th>
</tr>
</thead>
<tbody>
<tr>
<td>Degrees Fahrenheit</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

12. On Saturday, Querida gave away 416 coupons for a free appetizer at a local restaurant. The next day, she gave away about 52 coupons an hour.

   a. Is the number of coupons Querida gave away on Sunday proportional to the number of hours she worked that day?

<table>
<thead>
<tr>
<th>Hours Worked on Sunday</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coupons Given Away on Sunday</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

   b. Is the total number of coupons Querida gave away on Saturday and Sunday proportional to the number of hours she worked on Sunday?

<table>
<thead>
<tr>
<th>Hours Worked on Sunday</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coupons Given Away on Weekend</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

13. **MP Justify Conclusions** The fee for ride tickets at a carnival is shown in the table at the right.

   a. Is the fee for ride tickets proportional to the number of tickets? Explain your reasoning.

<table>
<thead>
<tr>
<th>Tickets</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fee ($)</td>
<td>5</td>
<td>9.50</td>
<td>13.50</td>
<td>16</td>
</tr>
</tbody>
</table>

   b. Can you determine the fee for 30 ride tickets? Explain.
14. Mr. Martinez is comparing the prices of oranges from several different markets. Determine whether each market uses proportional or nonproportional pricing. Place a checkmark in the column for the correct type of relationship.

<table>
<thead>
<tr>
<th>Number of Oranges</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Cost ($)</td>
<td>3.50</td>
<td>6.50</td>
<td>9.00</td>
<td>10.25</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Number of Oranges</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Cost ($)</td>
<td>3.25</td>
<td>6.50</td>
<td>9.75</td>
<td>13.00</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Number of Oranges</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Cost ($)</td>
<td>3.75</td>
<td>7.50</td>
<td>11.25</td>
<td>15.00</td>
</tr>
<tr>
<td></td>
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<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Number of Oranges</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Cost ($)</td>
<td>3.65</td>
<td>7.30</td>
<td>10.80</td>
<td>14.30</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>

15. A grocery store sells a 2.5-pound bag of mixed nuts for $9.25. The prices of mixed nuts are proportional. Determine if each of the following bags of mixed nuts could have been sold by the grocery store. Select yes or no.

a. 4.4 pounds for $16.28  ☐ Yes ☐ No
b. 3.2 pounds for $12  ☐ Yes ☐ No
c. 2.8 pounds for $10.50  ☐ Yes ☐ No

Common Core Spiral Review

Find the value of each expression if \( x = 12 \). 6.EE.2

16. \( 3x \)  
17. \( 2x - 4 \)  
18. \( 5x + 30 \)  
19. \( 3x - 2x \)  
20. \( x - 12 \)  
21. \( \frac{x}{4} \)

Make a table to solve the situation. 6.RP.3a

22. Brianna downloads 9 songs each month onto her MP3 player. Show the total number of songs downloaded after 1, 2, 3, and 4 months.

<table>
<thead>
<tr>
<th>Month</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Songs</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

40  Need more practice? Download more Extra Practice at connectED.mcgraw-hill.com.
Case #1 'Round and 'Round
The Forte family visited the Mall of America in Minneapolis. The Ferris wheel in the mall's amusement park is about 22.5 meters tall.

What is the approximate height of the Mall of America Ferris wheel in feet if 1 foot is about 0.3 meter?

In mathematics, there is a four-step problem-solving plan you can use to help you solve any problem. The four steps are Understand, Plan, Solve, and Check.

1 Understand What are the facts?
- The Mall of America Ferris wheel is about 22.5 meters tall.
- You need to find the height of the Ferris wheel in feet.

2 Plan What is your strategy to solve this problem?
To solve the problem, write an expression that converts meters to feet. Then divide out common units.

3 Solve How can you apply the strategy?
One foot is about 0.3 meter. Convert 22.5 meters to feet.

\[ 22.5 \text{ meters} \cdot \frac{1 \text{ foot}}{0.3 \text{ meter}} \approx \frac{22.5}{0.3} \text{ or } 75 \text{ feet} \]

So, the Ferris wheel is about 75 feet tall.

4 Check Does the answer make sense?
There is a little more than 3 feet in a meter.

Since 3 \cdot 22.5 is 67.5 and 75 feet is a little more than 67.5 feet, the answer is reasonable.

Analyze the Strategy

Reason Inductively Explain in your own words how the four-step plan helps you solve real-world problems.
Case #2 Cool Treats

Mr. Marino’s class learned the average American consumes about 23 quarts of ice cream every year. The class also learned the average American in the north-central United States consumes about 19 quarts more.

How much ice cream in gallons is consumed every year by the average American in the north-central United States?

1 Understand

Read the problem. What are you being asked to find?

I need to find

Fill in each box with the information you know.

The average American consumes about ___ quarts of ice cream.

The average American in the north-central United States consumes about ___ quarts more.

2 Plan

Choose two operations to solve the problem.

I will

3 Solve

How will you use the operations?

I will

Find total quarts. Convert to gallons.

___ quarts + ___ quarts = ___ quarts

___ quarts • \( \frac{1\text{ gallon}}{4\text{ quarts}} \) = ___ gallons

The average American in the north-central United States consumes about ___ gallons of ice cream each year.

4 Check

Use information from the problem to check your solution.
Work with a small group to solve the following cases. Show your work on a separate piece of paper.

**Case #3  Financial Literacy**
Terry opened a savings account in December with $150 and deposited $30 each month beginning in January.

What is the value of Terry's account at the end of July?

**Case #4  STEM**
About how many centimeters longer is the average femur than the average tibia? (Hint: 1 inch ≈ 2.54 centimeters)

<table>
<thead>
<tr>
<th>Bones in a Human Leg</th>
<th>Length (in.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Femur (upper leg)</td>
<td>19.88</td>
</tr>
<tr>
<td>Tibia (inner lower leg)</td>
<td>16.94</td>
</tr>
<tr>
<td>Fibula (outer lower leg)</td>
<td>15.94</td>
</tr>
</tbody>
</table>

**Case #5  Patterns**
Numbers that can be represented by a triangular arrangement of dots are called *triangular numbers*. The first four triangular numbers are shown.

Describe the pattern in the first four numbers. Then list the next three triangular numbers.

**Case #6  Transportation**
Mr. Norman has agreed to drive 4 students to their gymnastics practice.

If one student rides in the front seat and three students ride in the back, in how many ways can the 4 students be arranged in the car?
Vocabulary Check

1. **Be Precise** Define *complex fraction*. Give two examples of a complex fraction. (Lesson 2)

2. Fill in the blank in the sentence below with the correct term. (Lesson 1)
   When a rate is simplified so that it has a denominator of 1 unit, it is called a(n) ___________ rate.

Skills Check and Problem Solving

Find each unit rate. Round to the nearest hundredth if necessary. (Lesson 7)

3. 750 yards in 25 minutes

4. $420 for 15 tickets

Simplify. (Lesson 7)

5. \(\frac{\frac{9}{1}}{\frac{3}{1}} = \)

6. \(\frac{\frac{2}{4}}{\frac{4}{4}} = \)

7. \(\frac{\frac{1}{6}}{\frac{1}{6}} = \)

8. A tourist information center charges $10 per hour to rent a bicycle. Is the rental charge proportional to the number of hours you rent the bicycle? Justify your response. (Lesson 4)

9. **Persevere with Problems** A cruise ship is traveling at a speed of 20 knots. A knot is approximately equal to 1.151 miles per hour. What is the approximate speed the ship is traveling in yards per second? Round to the nearest tenth. (Lesson 3)
Maps have grids to locate cities. The **coordinate plane** is a type of grid that is formed when two number lines intersect at their zero points. The number lines separate the coordinate plane into four regions called **quadrants**.

An **ordered pair** is a pair of numbers, such as \((1, 2)\), used to locate or graph points on the coordinate plane.

The **x-coordinate** corresponds to a number on the **x-axis**. \((1, 2)\) The **y-coordinate** corresponds to a number on the **y-axis**.

Label the coordinate plane with the terms **ordered pair**, **x-coordinate**, and **y-coordinate**.

Graph points \((2, 3)\) and \((-3, -2)\) above. Connect the three points on the coordinate plane. Describe the graph.

---

**Which **MP** Mathematical Practices did you use?**

Shade the circle(s) that applies.

1. Persevere with Problems  
2. Reason Abstractly  
3. Construct an Argument  
4. Model with Mathematics  
5. Use Math Tools  
6. Attend to Precision  
7. Make Use of Structure  
8. Use Repeated Reasoning
Another way to determine whether two quantities are proportional is to graph the quantities on the coordinate plane. If the graph of the two quantities is a straight line through the origin, then the two quantities are proportional.

**Example**

1. The slowest mammal on Earth is the tree sloth. It moves at a speed of 6 feet per minute. Determine whether the number of feet the sloth moves is proportional to the number of minutes it moves by graphing on the coordinate plane. Explain your reasoning.

**Step 1** Make a table to find the number of feet walked for 0, 1, 2, 3, and 4 minutes.

<table>
<thead>
<tr>
<th>Time (min)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance (ft)</td>
<td>0</td>
<td>6</td>
<td>12</td>
<td>18</td>
<td>24</td>
</tr>
</tbody>
</table>

**Step 2** Graph the ordered pairs (time, distance) on the coordinate plane. Then connect the ordered pairs.

The line passes through the origin and is a straight line. So, the number of feet traveled is proportional to the number of minutes.

**Got it?** Do this problem to find out.

a. James earns $5 an hour babysitting. Determine whether the amount of money James earns babysitting is proportional to the number of hours he babysits by graphing on the coordinate plane. Explain your reasoning in the work zone.
Example

2. The cost of renting video games from Games Inc. is shown in the table. Determine whether the cost is proportional to the number of games rented by graphing on the coordinate plane. Explain your reasoning.

<table>
<thead>
<tr>
<th>Number of Games</th>
<th>Cost ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>7</td>
</tr>
<tr>
<td>4</td>
<td>9</td>
</tr>
</tbody>
</table>

**Step 1** Write the two quantities as ordered pairs (number of games, cost).

The ordered pairs are (1, 3), (2, 5), (3, 7), and (4, 9).

**Step 2** Graph the ordered pairs on the coordinate plane. Then connect the ordered pairs and extend the line to the y-axis.

The line does not pass through the origin. So, the cost of the video games is not proportional to the number of games rented.

**Check** The ratios are not constant $\frac{1}{3} \neq \frac{2}{5}$.

Quick Review

Remember that the independent variable is the input and the dependent variable is the output. When drawing a graph, include the labels for both axes.

Got it? Do this problem to find out.

b. The table shows the number of Calories an athlete burned per minute of exercise. Determine whether the number of Calories burned is proportional to the number of minutes by graphing on the coordinate plane. Explain your reasoning in the Work Zone.

<table>
<thead>
<tr>
<th>Calories Burned</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Minutes</td>
</tr>
<tr>
<td>0</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
</tr>
</tbody>
</table>

Lesson 5 Graph Proportional Relationships
Example

3. Which batting cage represents a proportional relationship between the number of pitches thrown and the cost? Explain.

The graph for Softball Plus is a straight line, but it does not pass through the origin. So, the relationship is not proportional.

The graph for the Fun Center is a straight line through the origin. So, the relationship between the number of the pitches thrown and the cost is proportional.

Guided Practice

1. The cost of 3-D movie tickets is $12 for 1 ticket, $24 for 2 tickets, and $36 for 3 tickets. Determine whether the cost is proportional to the number of tickets by graphing on the coordinate plane. Explain your reasoning. (Examples 1 and 2)

2. The number of books two stores sell after 1, 2, and 3 days is shown. Which book sale represents a proportional relationship between time and books? Explain. (Example 3)

Rate Yourself!

How confident are you about identifying proportional relationships by graphing? Check the box that applies.

For more help, go online to access a Personal Tutor.
Model with Mathematics: Determine whether the relationship between the two quantities shown in each table are proportional by graphing on the coordinate plane. Explain your reasoning. (Examples 1 and 2)

1. Savings Account

<table>
<thead>
<tr>
<th>Week</th>
<th>Account Balance ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>125</td>
</tr>
<tr>
<td>2</td>
<td>150</td>
</tr>
<tr>
<td>3</td>
<td>175</td>
</tr>
</tbody>
</table>

2. Calories in Fruit Cups

<table>
<thead>
<tr>
<th>Servings</th>
<th>Calories</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>70</td>
</tr>
<tr>
<td>3</td>
<td>210</td>
</tr>
<tr>
<td>5</td>
<td>350</td>
</tr>
</tbody>
</table>

3. The height of two plants is recorded after 1, 2, and 3 weeks as shown in the graph at the right. Which plants' growth represents a proportional relationship between time and height? Explain. (Example 3)
4. The perimeter of a square is 4 times as great as the length of any of its sides. Determine if the perimeter of a square is proportional to its side length. Explain.

5. A health club charges $35 a month for membership fees. Determine whether the cost of membership is proportional to the number of months. Explain your reasoning.

H.O.T. Problems  Higher Order Thinking

6. M Reason Abstractly Describe some data that when graphed would represent a proportional relationship. Explain your reasoning.

7. M Persevere with Problems The greenhouse temperatures at certain times are shown in the table. The greenhouse maintains temperatures between 65°F and 85°F. Suppose the temperature increases at a constant rate. Create a graph of the time and temperatures at each hour from 1:00 P.M. to 8:00 P.M. Is the relationship proportional? Explain.

<table>
<thead>
<tr>
<th>Time</th>
<th>Temperature (°F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1:00 P.M.</td>
<td>66</td>
</tr>
<tr>
<td>6:00 P.M.</td>
<td>78.5</td>
</tr>
<tr>
<td>8:00 P.M.</td>
<td>83.5</td>
</tr>
</tbody>
</table>

8. M Model with Mathematics Write a real-world problem that describes a proportional relationship. Make a table of values and graph the ordered pairs on the coordinate plane.
Extra Practice

Determine whether the relationship between the two quantities shown in each table are proportional by graphing on the coordinate plane. Explain your reasoning.

9. **Cooling Water**

<table>
<thead>
<tr>
<th>Time (min)</th>
<th>Temperature (°F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>95</td>
</tr>
<tr>
<td>10</td>
<td>90</td>
</tr>
<tr>
<td>15</td>
<td>85</td>
</tr>
</tbody>
</table>

Not proportional; The graph does not pass through the origin.

10. **Pizza Recipe**

<table>
<thead>
<tr>
<th>Number of Pizzas</th>
<th>Cheese (oz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8</td>
</tr>
<tr>
<td>4</td>
<td>32</td>
</tr>
<tr>
<td>7</td>
<td>56</td>
</tr>
</tbody>
</table>

Copy and Solve  Determine if each situation represents a proportional relationship. Graph on a separate piece of paper. Write an explanation for each situation.

11. **Justify Conclusions** An airplane is flying at an altitude of 4,000 feet and descends at a rate of 200 feet per minute. Determine whether the altitude is proportional to the number of minutes. Explain your reasoning.

12. Frank and Allie purchased cell phone plans through different providers. Their costs for several minutes are shown. Graph each plan to determine whose plan is proportional to the number of minutes the phone is used. Explain your reasoning.

<table>
<thead>
<tr>
<th>Time (min)</th>
<th>Frank's Cost ($)</th>
<th>Allie's Cost ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>4.00</td>
</tr>
<tr>
<td>3</td>
<td>1.50</td>
<td>4.50</td>
</tr>
<tr>
<td>6</td>
<td>3.00</td>
<td>5.00</td>
</tr>
</tbody>
</table>
13. The relationship between the number of heartbeats and the time shown in the graph is proportional. Determine if each ordered pair represents a point from this relationship. Select yes or no.
   a. (5, 10) □ Yes □ No
   b. (14, 7) □ Yes □ No
   c. (8, 16) □ Yes □ No

14. The table shows the rental costs for a moving truck.

<table>
<thead>
<tr>
<th>Miles Driven</th>
<th>50</th>
<th>100</th>
<th>150</th>
<th>200</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Cost ($)</td>
<td>40</td>
<td>60</td>
<td>80</td>
<td>100</td>
</tr>
</tbody>
</table>

Graph the data on the coordinate plane and explain whether the relationship between the number of miles and the total cost is proportional or nonproportional.

---

**CCSS Common Core Spiral Review**

Write each ratio as a fraction in simplest form. 6.RP.1

15. A class has 10 boys and 15 girls. What is the ratio of boys to girls?

16. A car dealership has 55 cars and 11 vans. What is the ratio of cars to vans?

17. A drawer has 4 red shirts and 8 green shirts. What is the ratio of red shirts to the total number of shirts?

18. A store sells 13 coffees and 65 hot chocolates. What is the ratio of coffees to hot chocolates?
HOW are proportional and nonproportional linear relationships alike? HOW are they different?

Albert and Bianca joined an online discussion group. Each student posted four comments. The number of replies to each of their comments is shown in the table. Determine if each data set represents a proportional relationship.

### Hands-On Activity

**Step 1**
Arrange centimeter cubes to model the number of replies per comment as shown in the diagram below.

<table>
<thead>
<tr>
<th>Student</th>
<th>Albert</th>
<th>Bianca</th>
</tr>
</thead>
<tbody>
<tr>
<td>Comment Number</td>
<td>1 2 3 4</td>
<td>1 2 3 4</td>
</tr>
<tr>
<td>Number of Replies</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Step 2**
Complete each table. Then graph the data on the coordinate plane. You may wish to use a different color pencil for each data set.

#### Albert's Comments

<table>
<thead>
<tr>
<th>Comment Number (x)</th>
<th>Number of Replies (y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
</tbody>
</table>

#### Bianca's Comments

<table>
<thead>
<tr>
<th>Comment Number (x)</th>
<th>Number of Replies (y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
</tbody>
</table>
1. Describe any patterns in the data.

2. Connect the ordered pairs with a straight line for each graph. Then describe the graphs.

3. Predict the next three points on the graph for each data.

4. Compare and contrast the relationship shown in each graph. What do you notice?

5. **Model with Mathematics** Use a table and graph to describe a real-world situation that represents a proportional relationship. Then explain how you could change your situation so that it represents a nonproportional relationship.

6. **Inquiry** How are proportional and nonproportional linear relationships alike? How are they different?
Fruit Smoothies: Katie and some friends want to buy fruit smoothies. They go to a health food store that advertises a sale of 2 fruit smoothies for $5.

1. Fill in the boxes to write a ratio that compares the cost to the number of fruit smoothies.

\[
\frac{\Box}{\Box} \quad \text{smoothies}
\]

2. Suppose Katie and her friends buy 6 fruit smoothies. Complete the ratio that compares the cost to the number of fruit smoothies.

\[
\frac{\Box}{6 \text{ smoothies}}
\]

3. Is the cost proportional to the number of fruit smoothies for two and six smoothies? Explain.

Which Mathematical Practices did you use? Shade the circle(s) that applies.

1. Persevere with Problems
2. Reason Abstractly
3. Construct an Argument
4. Model with Mathematics
5. Use Math Tools
6. Attend to Precision
7. Make Use of Structure
8. Use Repeated Reasoning
Key Concept

Write and Solve Proportions

Words
A proportion is an equation stating that two ratios or rates are equivalent.

Numbers
\( \frac{5}{3} = \frac{4}{6} \)

Algebra
\( \frac{a}{b} = \frac{c}{d}, b \neq 0, d \neq 0 \)

Consider the following proportion.

\[
\frac{a}{b} \cdot \frac{1}{d} = \frac{c}{d} \cdot \frac{1}{b}
\]

Multiply each side by \(bd\) and divide out common factors.

\[ad = bc\]
Simplify.

The products \(ad\) and \(bc\) are called the cross products of this proportion. The cross products of any proportion are equal.

\[
\begin{align*}
6 \cdot 3 &= 8 \cdot 3 \\
8 \cdot 4 &= 6 \cdot 4 \\
\end{align*}
\]

\[= 24 \quad = 24\]

Real World

Example

1. After 2 hours, the air temperature had risen 7°F. Write and solve a proportion to find the amount of time it will take at this rate for the temperature to rise an additional 13°F.

Write a proportion. Let \(t\) represent the time in hours.

\[
\frac{7}{2} = \frac{13}{t}
\]

Multiply each side by 7.

\[
7t = 26
\]
Divide each side by 7.

\[
t \approx 3.7
\]
Simplify.

It will take about 3.7 hours to rise an additional 13°F.

Got it? Do these problems to find out.

Solve each proportion.

a. \( \frac{x}{4} = \frac{9}{10} \)
b. \( \frac{2}{34} = \frac{5}{y} \)
c. \( \frac{7}{3} = \frac{n}{21} \)
Example

2. If the ratio of Type O to non-Type O donors at a blood drive was 37:43, how many donors would be Type O, out of 300 donors?

\[
\text{Type O donors} \rightarrow \frac{37}{37 + 43} = \frac{37}{80}
\]

Write a proportion. Let \( t \) represent the number of Type O donors.

\[
\frac{37}{80} = \frac{t}{300}
\]

Find the cross products.

\[
37 \cdot 300 = 80t
\]

Multiply.

\[
11,100 = 80t
\]

Divide each side by 80.

\[
\frac{11,100}{80} = \frac{80t}{80}
\]

Simplify.

\[
138.75 = t
\]

There would be about 139 Type O donors.

Got it? Do this problem to find out.

d. The ratio of 7th grade students to 8th grade students in a soccer league is 17:23. If there are 200 students in all, how many are in the 7th grade?

Use Unit Rate

You can also use the unit rate to write an equation expressing the relationship between two proportional quantities.

Examples

3. Olivia bought 6 containers of yogurt for $7.68. Write an equation relating the cost \( c \) to the number of yogurts \( y \). How much would Olivia pay for 10 yogurts at this same rate?

\[
\frac{\text{cost in dollars}}{\text{containers of yogurt}} = \frac{7.68}{6} \text{ or } 1.28 \text{ per container}
\]

The cost is $1.28 times the number of containers of yogurt.

\[
c = 1.28y
\]

Let \( c \) represent the cost, let \( y \) represent the number of yogurts.

\[
= 1.28(10)
\]

Replace \( y \) with 10.

\[
= 12.80
\]

Multiply.

The cost for 10 containers of yogurt is $12.80.
4. Jaycee bought 8 gallons of gas for $31.12. Write an equation relating the cost $c$ to the number of gallons $g$ of gas. How much would Jaycee pay for 11 gallons at this same rate?

Find the unit rate between cost and gallons.

\[
\frac{\text{cost in dollars}}{\text{gasoline in gallons}} = \frac{31.12}{8} \text{ or } 3.89 \text{ per gallon}
\]

The cost is $3.89 times the number of gallons.

\[
c = 3.89g\quad \text{Let } c \text{ represent the cost. Let } g \text{ represent the number of gallons.}
\]

\[
= 3.89(11)\quad \text{Replace } g \text{ with 11.}
\]

\[
= 42.79\quad \text{Multiply.}
\]

The cost for 11 gallons of gas is $42.79.

Got it? Do this problem to find out.

e. Olivia typed 2 pages in 15 minutes. Write an equation relating the number of minutes $m$ to the number of pages $p$ typed. How long will it take her to type 10 pages at this rate?

Guided Practice

Solve each proportion. (Examples 1 and 2)

1. \[\frac{k}{7} = \frac{32}{56}\quad k = \quad \]

2. \[\frac{32}{9} = \frac{n}{36}\quad n = \quad \]

3. \[\frac{41}{x} = \frac{5}{2}\quad x = \quad \]

4. Trina earns $28.50 tutoring for 3 hours. Write an equation relating her earnings $m$ to the number of hours $h$ she tutors. Assuming the situation is proportional, how much would Trina earn tutoring for 2 hours? for 4.5 hours? (Examples 3 and 4)

Rate Yourself!

How confident are you about solving proportions? Check the box that applies.

For more help, go online to access a Personal Tutor.

Building on the Essential Question How do you solve a proportion?
Solve each proportion. (Examples 1 and 2)

1. \[ \frac{15}{6} = \frac{10}{p} \quad p = \]  
2. \[ \frac{44}{p} = \frac{11}{5} \quad p = \]  
3. \[ \frac{2}{w} = \frac{0.4}{0.7} \quad w = \]

Assume the situations are proportional. Write and solve by using a proportion. (Examples 1 and 2)

4. Evarado paid $1.12 for a dozen eggs at his local grocery store. Determine the cost of 3 eggs.

5. Sheila mixed 3 ounces of blue paint with 2 ounces of yellow paint. She decided to create 20 ounces of the same mixture. How many ounces of yellow paint does Sheila need for the new mixture?

Assume the situations are proportional. Use the unit rate to write an equation, then solve. (Examples 3 and 4)

6. A car can travel 476 miles on 14 gallons of gas. Write an equation relating the distance \( d \) to the number of gallons \( g \). How many gallons of gas does this car need to travel 578 miles?

7. Mrs. Baker paid $2.50 for 5 pounds of bananas. Write an equation relating the cost \( c \) to the number of pounds \( p \) of bananas. How much would Mrs. Baker pay for 8 pounds of bananas?

8. A woman who is 64 inches tall has a shoulder width of 16 inches. Write an equation relating the height \( h \) to the width \( w \). Find the height of a woman who has a shoulder width of 18.5 inches.
9. At an amusement park, 360 visitors rode the roller coaster in 3 hours. Write and solve a proportion to find the number of visitors at this rate who will ride the roller coaster in 7 hours. (Examples 3 and 4)

10. **Reason Abstractly** Use the table to write a proportion relating the weights on two planets. Then find the missing weight. Round to the nearest tenth.
   a. Earth: 90 pounds; Venus: _____ pounds
   b. Mercury: 55 pounds; Earth: _____ pounds
   c. Jupiter: 350 pounds; Uranus: _____ pounds
   d. Venus: 115 pounds; Mercury: _____ pounds

   **Weights on Different Planets**
   - Earth Weight = 120 pounds
   - Mercury: 45.6 pounds
   - Venus: 109.2 pounds
   - Uranus: 96 pounds
   - Jupiter: 304.8 pounds

11. **Justify Conclusions** A powdered drink mix calls for a ratio of powder to water of 1:8. If there are 32 cups of powder, how many total cups of water are needed? Explain your reasoning.

12. \( \frac{2}{3} = \frac{18}{x+5} \)  
13. \( \frac{x-4}{10} = \frac{7}{5} \)  
14. \( \frac{4.5}{17-x} = \frac{3}{8} \)

15. **Justify Conclusions** A rectangle has an area of 36 square units. As the length and the width change, what do you know about their product? Is the length proportional to the width? Justify your reasoning.

<table>
<thead>
<tr>
<th>Rectangle</th>
<th>Length</th>
<th>Width</th>
<th>Area (units²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>3</td>
<td>12</td>
<td>36</td>
</tr>
<tr>
<td>B</td>
<td>6</td>
<td>6</td>
<td>36</td>
</tr>
<tr>
<td>C</td>
<td>9</td>
<td>4</td>
<td>36</td>
</tr>
</tbody>
</table>
Solve each proportion.

16. \( \frac{x}{13} = \frac{18}{39} \) \( x = \_ \)

\[
\times \cdot 39 = 13 \cdot 18 \\
39x = 234 \\
\frac{39x}{39} = \frac{234}{39} \\
x = 6
\]

17. \( \frac{6}{25} = \frac{d}{30} \) \( d = \_ \)

18. \( \frac{2.5}{6} = \frac{h}{9} \) \( h = \_ \)

Assume the situations are proportional. Write and solve by using a proportion.

19. For every person who has the flu, there are 6 people who have only flu-like symptoms. If a doctor sees 40 patients, determine approximately how many patients you would expect to have only flu-like symptoms.

20. For every left-handed person, there are about 4 right-handed people. If there are 30 students in a class, predict the number of students who are right-handed.

21. Jeremiah is saving money from a tutoring job. After the first three weeks, he saved $135. Assume the situation is proportional. Use the unit rate to write an equation relating the amount saved \( s \) to the number of weeks \( w \) worked. At this rate, how much will Jeremiah save after eight weeks?

22. **Make a Prediction** A speed limit of 100 kilometers per hour (kph) is approximately equal to 62 miles per hour (mph). Write an equation relating kilometers per hour \( k \) to miles per hour \( m \). Then predict the following measures. Round to the nearest tenth.

a. a speed limit in mph for a speed limit of 75 kph

b. a speed limit in kph for a speed limit of 20 mph
23. Part of Nicole's pumpkin muffin recipe is shown. How many cups of flour are needed to make 5 dozen muffins?

24. An amusement park line for passengers waiting to ride a rollercoaster is moving about 16 feet every 10 minutes. Jason and his friends are standing 40 feet from the front of the line. Select values to set up a proportion to represents this situation.

\[
\frac{16}{10} = \frac{40}{x}
\]

Solve the proportion to determine how long it will take for Jason and his friends to reach the front of the line.

25. The table shows the cost to have various numbers of pizzas delivered from Papa's Slice of Italy pizzeria. Is the relationship between the cost and the number of pizzas proportional? Explain. 7.RP.2a

<table>
<thead>
<tr>
<th>Number of Pizzas</th>
<th>Cost ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>12.50</td>
</tr>
<tr>
<td>2</td>
<td>20</td>
</tr>
<tr>
<td>3</td>
<td>27.50</td>
</tr>
<tr>
<td>4</td>
<td>35</td>
</tr>
</tbody>
</table>

26. Brenna charges $15, $30, $45, and $60 for babysitting 1, 2, 3, and 4 hours, respectively. Is the relationship between the amount charged and the number of hours proportional? If so, find the unit rate. If not, explain why not. 7.RP.2a

Find each unit rate. 6.RP.3b

27. 50 miles on 2.5 gallons

28. 2,500 kilobytes in 5 minutes
**Inquiry**

**HOW is unit rate related to rate of change?**

Happy Hound is a doggie daycare where people drop off their dogs while they are at work. It costs $3 for 1 hour, $6 for 2 hours, and $9 for 3 hours of doggie daycare. Farah takes her dog to Happy Hound several days a week. Farah wants to determine if the number of hours of daycare is related to the cost.

**Hands-On Activity**

**Step 1**

Assume the pattern in the table continues. Complete the table shown.

<table>
<thead>
<tr>
<th>Number of Hours</th>
<th>Cost ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>9</td>
</tr>
</tbody>
</table>

**Step 2**

The cost depends on the number of hours. So, the cost is the output \( y \), and the number of hours is the \( x \).

Graph the data on the coordinate plane below.
Investigate

Refer to the Investigation. Work with a partner.

1. **MP Justify Conclusions** Describe the graph.

2. What is the cost per hour, or unit rate, charged by Happy Hound?

3. Use the graph to examine any two consecutive points. By how much does y change? By how much does x change?

4. The first two ordered pairs on the graph are (1, 3) and (2, 6). You can find the rate of change by writing the ratio of the change in y to the change in x.

   Find the rate of change shown in the graph.

Analyze and Reflect

Work with a partner to answer the following question.

5. Pampered Pooch charges $5 for 1 hour of doggie daycare, $10 for 2 hours, and $15 for 3 hours.

   a. What is the unit rate?

   b. What is the rate of change?

   c. **MP Reason Inductively** How do the rates of change for doggie daycare at Pampered Pooch and Happy Hound compare?

Create

6. **MP Model with Mathematics** Describe a doggie daycare situation that has a rate of change less than that of Happy Hound.

7. **Inquiry** HOW is unit rate related to rate of change?
Vocabulary Start-Up

A rate of change is a rate that describes how one quantity changes in relation to another. In a linear relationship, the rate of change between any two quantities is the same. A linear relationship has a constant rate of change.

Real-World Link

A computer programmer charges customers per line of code written. Fill in the blanks with the amount of change between consecutive numbers.

<table>
<thead>
<tr>
<th>Lines of Code</th>
<th>50</th>
<th>100</th>
<th>150</th>
<th>200</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost ($)</td>
<td>1,000</td>
<td>2,000</td>
<td>3,000</td>
<td>4,000</td>
</tr>
</tbody>
</table>

Label the diagram below with the terms change in lines, change in dollars, and constant rate of change.

\[
\text{Cost} \quad \frac{1,000}{50 \text{ lines}} = \text{Cost per line}
\]

The cost per line of programming code is $20.

Which MP Mathematical Practices did you use?
Shade the circle(s) that applies.

1. Persevere with Problems
2. Reason Abstractly
3. Construct an Argument
4. Model with Mathematics
5. Use Math Tools
6. Attend to Precision
7. Make Use of Structure
8. Use Repeated Reasoning
Use a Table

You can use a table to find a constant rate of change.

**Example**

1. The table shows the amount of money a booster club makes washing cars for a fundraiser. Use the information to find the constant rate of change in dollars per car.

<table>
<thead>
<tr>
<th>Cars Washed</th>
<th>Number</th>
<th>Money ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>5</td>
<td>40</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>80</td>
</tr>
<tr>
<td></td>
<td>15</td>
<td>120</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>160</td>
</tr>
</tbody>
</table>

Find the unit rate to determine the constant rate of change.

\[
\text{change in money \over change in cars} = \frac{40 \text{ dollars}}{5 \text{ cars}} = \frac{8 \text{ dollars}}{1 \text{ car}}
\]

The money earned increases by $40 for every 5 cars. Write as a unit rate.

So, the number of dollars earned increases by $8 for every car washed.

**Got it? Do these problems to find out.**

a. The table shows the number of miles a plane traveled while in flight. Use the information to find the approximate constant rate of change in miles per minute.

<table>
<thead>
<tr>
<th>Time (min)</th>
<th>30</th>
<th>60</th>
<th>90</th>
<th>120</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance (mi)</td>
<td>290</td>
<td>580</td>
<td>870</td>
<td>1,160</td>
</tr>
</tbody>
</table>

b. The table shows the number of students that buses can transport. Use the table to find the constant rate of change in students per school bus.

<table>
<thead>
<tr>
<th>Number of Buses</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Students</td>
<td>144</td>
<td>216</td>
<td>288</td>
<td>360</td>
</tr>
</tbody>
</table>
Use a Graph

You can also use a graph to find a constant rate of change and to analyze points on the graph.

**Examples**

2. The graph represents the distance traveled while driving on a highway. Find the constant rate of change.

   To find the rate of change, pick any two points on the line, such as (0, 0) and (1, 60).

   \[
   \frac{\text{change in miles}}{\text{change in hours}} = \frac{(60 - 0) \text{ miles}}{(1 - 0) \text{ hours}} = \frac{60 \text{ miles}}{1 \text{ hour}}
   \]

3. Explain what the points (0, 0) and (1, 60) represent.

   The point (0, 0) represents traveling zero miles in zero hours. The point (1, 60) represents traveling 60 miles in 1 hour. Notice that this is the constant rate of change.

**Got it?** Do these problems to find out.

c. Use the graph to find the constant rate of change in miles per hour while driving in the city.

d. On the lines below, explain what the points (0, 0) and (1, 30) represent.
Example

4. The table and graph below show the hourly charge to rent a bicycle at two different stores. Which store charges more per bicycle? Explain.

<table>
<thead>
<tr>
<th>Pedals Rentals</th>
<th>Super Cycles</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time (hour)</td>
<td>Cost ($)</td>
</tr>
<tr>
<td>2</td>
<td>24</td>
</tr>
<tr>
<td>3</td>
<td>36</td>
</tr>
<tr>
<td>4</td>
<td>48</td>
</tr>
</tbody>
</table>

The cost at Pedals Rentals increases by $12 every hour. The cost at Super Cycles increases by $8 every hour.

So, Pedals Rentals charges more per hour to rent a bicycle.

Guided Practice

1. The table and graph below show the amount of money Mi-Ling and Daniel save each week. Who saves more each week? Explain. (Examples 1, 2, and 4)

<table>
<thead>
<tr>
<th>Mi-Ling's Savings</th>
<th>Daniel's Savings</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time (weeks)</td>
<td>Time (weeks)</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>4</td>
</tr>
</tbody>
</table>

2. Refer to the graph in Exercise 1. Explain what the points (0, 0) and (1, 10) represent. (Example 3)

3. Building on the Essential Question How can you find the unit rate on a graph that goes through the origin?

Rate Yourself!

Are you ready to move on? Shade the section that applies.

- I have a few questions. I'm ready to move on.
- I have a lot of questions.

For more help, go online to access a Personal Tutor.

68 Chapter 1 Ratios and Proportional Reasoning
Find the constant rate of change for each table. (Example 1)

<table>
<thead>
<tr>
<th>Time (s)</th>
<th>Distance (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>2</td>
<td>12</td>
</tr>
<tr>
<td>3</td>
<td>18</td>
</tr>
<tr>
<td>4</td>
<td>24</td>
</tr>
</tbody>
</table>

2. | Items | Cost ($) |
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>18</td>
</tr>
<tr>
<td>4</td>
<td>36</td>
</tr>
<tr>
<td>6</td>
<td>54</td>
</tr>
<tr>
<td>8</td>
<td>72</td>
</tr>
</tbody>
</table>

The graph shows the cost of purchasing T-shirts. Find the constant rate of change for the graph. Then explain what points (0, 0) and (1, 9) represent. (Examples 2 and 3)

4. The Guzman and Hashimoto families each took a 4-hour road trip. The distances traveled by each family are shown in the table and graph below. Which family averaged fewer miles per hour? Explain. (Example 4)

<table>
<thead>
<tr>
<th>Guzman's Road Trip</th>
<th>Hashimoto's Road Trip</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time (hours)</td>
<td>Distance (miles)</td>
</tr>
<tr>
<td>2</td>
<td>90</td>
</tr>
<tr>
<td>3</td>
<td>135</td>
</tr>
<tr>
<td>4</td>
<td>180</td>
</tr>
</tbody>
</table>

5. At 1:00 P.M., the water level in a pool is 13 inches. At 1:30 P.M., the water level is 18 inches. At 2:30 P.M., the water level is 28 inches. What is the constant rate of change?
Model with Mathematics  Refer to the lap times for Exercises a and b.

Let's calculate Seth's times.

<table>
<thead>
<tr>
<th>Lap</th>
<th>4</th>
<th>8</th>
<th>12</th>
<th>16</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance(m)</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>Time(s)</td>
<td>57.1</td>
<td>114.2</td>
<td>171.3</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

a. How long does it take Seth to race 1 mile? Write the constant rate of change in miles per second. Round to the nearest hundredth.

b. Graph the ordered pairs (time, distance) on the coordinate plane at the right. Connect the points with a solid line.

H.O.T. Problems

7. Model with Mathematics  Make a table where the constant rate of change is 6 inches for every foot.

<table>
<thead>
<tr>
<th>Feet</th>
<th>Inches</th>
</tr>
</thead>
</table>

8. Justify Conclusions  The terms in sequence A increase by 3. The terms in sequence B increase by 8. In which sequence do the terms form a steeper line when graphed as points on a coordinate plane? Justify your reasoning.

9. Persevere with Problems  The constant rate of change for the relationship shown in the table is $8 per hour. Find the missing values.

<table>
<thead>
<tr>
<th>Time (h)</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Earnings ($)</td>
<td>x</td>
<td>y</td>
<td>z</td>
</tr>
</tbody>
</table>

x = y = z =
Extra Practice

Find the constant rate of change for each table.

10.  
<table>
<thead>
<tr>
<th>Time (h)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wage ($)</td>
<td>0</td>
<td>9</td>
<td>18</td>
<td>27</td>
</tr>
</tbody>
</table>

\[
\text{\$9 per hour} \quad \frac{\text{change in wages}}{\text{change in hours}} = \frac{\$9}{1 \text{ hour}}
\]

11.  
<table>
<thead>
<tr>
<th>Minutes</th>
<th>1,000</th>
<th>1,500</th>
<th>2,000</th>
<th>2,500</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost ($)</td>
<td>38</td>
<td>53</td>
<td>68</td>
<td>83</td>
</tr>
</tbody>
</table>

12. Use the graph to find the constant rate of change. Then, explain what the points (0, 0) and (6, 72) represent.

13. **Justify Conclusions** Ramona and Josh earn money by babysitting. The amounts earned for one evening are shown in the table and graph. Who charged more per hour? Explain.

<table>
<thead>
<tr>
<th>Ramona's Earnings</th>
<th>Josh's Earnings</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time (hours)</td>
<td>Earnings ($)</td>
</tr>
<tr>
<td>2</td>
<td>18</td>
</tr>
<tr>
<td>3</td>
<td>27</td>
</tr>
<tr>
<td>4</td>
<td>36</td>
</tr>
</tbody>
</table>

14. The cost of 1 movie ticket is $7.50. The cost of 2 movie tickets is $15. Based on this constant rate of change, what is the cost of 4 movie tickets?
15. Reggie started a running program to prepare for track season. He ran a half hour each morning for 60 days. He averaged 6.5 miles per hour. What is the total number of miles Reggie ran over the 60-day period?

16. Select the correct constant rate of change for each table of data.

<table>
<thead>
<tr>
<th>Number of Apples</th>
<th>3</th>
<th>7</th>
<th>11</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Seeds</td>
<td>30</td>
<td>70</td>
<td>110</td>
</tr>
<tr>
<td>Number of Tables</td>
<td>4</td>
<td>6</td>
<td>9</td>
</tr>
<tr>
<td>Number of Chairs</td>
<td>48</td>
<td>72</td>
<td>108</td>
</tr>
<tr>
<td>Number of Passengers</td>
<td>24</td>
<td>60</td>
<td>120</td>
</tr>
<tr>
<td>Number of Vans</td>
<td>2</td>
<td>5</td>
<td>10</td>
</tr>
<tr>
<td>Number of Booklets</td>
<td>20</td>
<td>50</td>
<td>100</td>
</tr>
<tr>
<td>Number of Pages</td>
<td>2</td>
<td>5</td>
<td>10</td>
</tr>
</tbody>
</table>

**Common Core Spiral Review**

Write the output for each given input in the tables below. 5.OA.3

17. | Input | Add 4 | Output |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
<td>7</td>
</tr>
<tr>
<td>3</td>
<td>7</td>
<td>8</td>
</tr>
<tr>
<td>4</td>
<td>8</td>
<td>9</td>
</tr>
</tbody>
</table>

18. | Input | Subtract 5 | Output |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>25</td>
<td>20</td>
</tr>
<tr>
<td>40</td>
<td>35</td>
<td>30</td>
</tr>
<tr>
<td>50</td>
<td>45</td>
<td>40</td>
</tr>
<tr>
<td>60</td>
<td>55</td>
<td>50</td>
</tr>
</tbody>
</table>

19. | Input | Multiply by 2 | Output |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>4</td>
<td>8</td>
<td>8</td>
</tr>
</tbody>
</table>

20. | Input | Divide by 3 | Output |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>6</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>9</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>12</td>
<td>4</td>
<td>4</td>
</tr>
</tbody>
</table>

72 **Need more practice?** Download more Extra Practice at connectED.mcgraw-hill.com.
Recycling Hero Comics prints on recycled paper. The table shows the total number of pounds of recycled paper that has been used each day during the month.

<table>
<thead>
<tr>
<th>Day of Month</th>
<th>Total Recycled (lbs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>36</td>
</tr>
<tr>
<td>5</td>
<td>60</td>
</tr>
<tr>
<td>6</td>
<td>72</td>
</tr>
<tr>
<td>7</td>
<td>84</td>
</tr>
<tr>
<td>12</td>
<td>144</td>
</tr>
</tbody>
</table>

1. Graph the ordered pairs on the coordinate plane.

2. Explain why the graph is linear.

3. Use two points to find the constant rate of change.
   - Point 1: [ ] change in pounds $\rightarrow$ [ ] pounds
   - Point 2: [ ] change in days $\rightarrow$ [ ] days

So, the constant rate of change is $\frac{24}{2}$ or [ ] pounds per day.

Which Mathematical Practices did you use? Shade the circle(s) that applies.

1. Persevere with Problems
2. Reason Abstractly
3. Construct an Argument
4. Model with Mathematics
5. Use Math Tools
6. Attend to Precision
7. Make Use of Structure
8. Use Repeated Reasoning
Slope is the rate of change between any two points on a line.

\[
\text{Slope} = \frac{\text{change in } y}{\text{change in } x} \quad \rightarrow \quad \text{vertical change}
\]

\[
= \frac{2}{1} \quad \text{or} \quad 2
\]

In a linear relationship, the vertical change (change in y-value) per unit of horizontal change (change in x-value) is always the same. This ratio is called the slope of the function. The constant rate of change, or unit rate, is the same as the slope of the related linear relationship.

The slope tells how steep the line is. The vertical change is sometimes called "rise" while the horizontal change is called "run." You can say that slope = \( \frac{\text{rise}}{\text{run}} \).

Count the number of units that make up the rise of the line in the graph shown above. Write this number for the numerator of the fraction below. Count the number of units that make up the run of the line. Write this number for the denominator of the fraction below.

\[
\frac{\text{rise}}{\text{run}} = \frac{\square}{\square}
\]

So, the slope of the line is \( \frac{3}{2} \).
Example

1. The table below shows the relationship between the number of seconds \( y \) it takes to hear thunder after a lightning strike and the miles \( x \) you are from the lightning. Graph the data and find the slope. Explain what the slope represents.

<table>
<thead>
<tr>
<th>Miles (( x ))</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Seconds (( y ))</td>
<td>0</td>
<td>5</td>
<td>10</td>
<td>15</td>
<td>20</td>
<td>25</td>
</tr>
</tbody>
</table>

\[
\text{slope} = \frac{\text{change in } y}{\text{change in } x} \quad \text{Definition of slope}
\]

\[
= \frac{25 - 15}{5 - 3} \quad \text{Use (3, 15) and (5, 25).}
\]

\[
= \frac{10}{2} \quad \text{seconds}
\]

\[
= 5 \quad \text{miles}
\]

\[
= \frac{5}{1} \quad \text{Simplify.}
\]

So, for every 5 seconds between a lightning flash and the sound of thunder, there is 1 mile between you and the lightning strike.

Got it? Do this problem to find out.

a. Graph the data about plant height for a science fair project. Then find the slope of the line. Explain what the slope represents in the work zone.

<table>
<thead>
<tr>
<th>Week</th>
<th>Plant Height (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.5</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>4.5</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
</tr>
</tbody>
</table>
Example

2. Renaldo opened a savings account. Each week he deposits $300. Draw a graph of the account balance versus time. Find the numerical value of the slope and interpret it in words.

The slope of the line is the rate at which the account balance \(\frac{300}{1}\) week rises, or

Got it? Do this problem to find out.

b. Jessica has a balance of $35 on her cell phone account. She adds $10 each week for the next four weeks. In the work zone, graph the account balance versus time. Find the numerical value of the slope and interpret it in words.

Guided Practice

1. The table at the right shows the number of small packs of fruit snacks \(y\) per box \(x\). Graph the data. Then find the slope of the line. Explain what the slope represents. (Examples 1 and 2)

<table>
<thead>
<tr>
<th>Boxes, (x)</th>
<th>3</th>
<th>5</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fruit Snacks, (y)</td>
<td>12</td>
<td>20</td>
<td>28</td>
</tr>
</tbody>
</table>

Rate Yourself!

How well do you understand slope? Circle the image.

For more help, go online to access a Personal Tutor.
1. The table shows the number of pages Adriano read in x hours. Graph the data. Then find the slope of the line. Explain what the slope represents.  

<table>
<thead>
<tr>
<th>Time (h)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of pages</td>
<td>50</td>
<td>100</td>
<td>150</td>
<td>200</td>
</tr>
</tbody>
</table>

2. Graph the data. Find the numerical value of the slope and interpret it in words.

<table>
<thead>
<tr>
<th>Number of Yards</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Feet</td>
<td>3</td>
<td>6</td>
<td>9</td>
</tr>
</tbody>
</table>

3. The graph shows the average speed of two cars on the highway.
   a. What does (2, 120) represent?
   b. What does (1.5, 67.5) represent?
   c. What does the ratio of the y-coordinate to the x-coordinate for each pair of points on the graph represent?
   d. What does the slope of each line represent?
   e. Which car is traveling faster? How can you tell from the graph?
4. **Multiple Representations** Complete the graphic organizer on slope.

<table>
<thead>
<tr>
<th>slope</th>
</tr>
</thead>
<tbody>
<tr>
<td>Words</td>
</tr>
<tr>
<td>Pictures</td>
</tr>
<tr>
<td>Numbers</td>
</tr>
</tbody>
</table>

---

**H.O.T. Problems** Higher Order Thinking

5. **Find The Error** Marisol is finding the slope of the line containing the points (3, 7) and (5, 10). Find her mistake and correct it.

   The slope between the two points (3, 7) and (5, 10) is found like this:
   
   \[
   \text{slope} = \frac{\text{rise}}{\text{run}} = \frac{5 - 3}{10 - 7} = \frac{2}{3}
   \]

6. **Persevere with Problems** Kaya is saving money at a rate of $30 per month. Eduardo is saving money at a rate of $35 per month. They both started saving at the same time. If you were to create a table of values and graph each function, what would be the slope of each graph?

7. **Reason Inductively** Without graphing, determine whether \(A(5, 1), B(1, 0),\) and \(C(3, 3)\) lie on the same line. Explain your reasoning.

8. **Model with Mathematics** Name two points on a line that has a slope of \(\frac{5}{8}\).
9. **Justify Conclusions** The table to the right shows the number of markers per box. Graph the data. Then find the slope of the line. Explain what the slope represents.

<table>
<thead>
<tr>
<th>Boxes</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Markers</td>
<td>8</td>
<td>16</td>
<td>24</td>
<td>32</td>
</tr>
</tbody>
</table>

Use $(1, 8)$ and $(2, 16)$.  
\[
slope = \frac{\text{change in } y}{\text{change in } x} = \frac{16 - 8}{2 - 1} = \frac{8}{1}
\]

So, there are 8 markers in every box.

10. The table shows the cost to rent a paddle boat from two businesses.

   a. What does $(1, 20)$ represent?

   b. What does $(2, 50)$ represent?

**Copy and Solve** For Exercises 11–14, draw a graph on a separate sheet of grid paper to find each slope. Then record each slope and interpret its meaning.

11. Joshua swims 25 meters in 1 minute. Draw a graph of meters swam versus time. Find the value of the slope and interpret it in words.

12. The table shows the amount Maggie earns for various numbers of hours she babysits. Graph the data. Then find the slope of the line. Explain what the slope represents.

<table>
<thead>
<tr>
<th>Number of Hours</th>
<th>Water Wheels Cost ($)</th>
<th>Fun in the Sun Cost ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>20</td>
<td>25</td>
</tr>
<tr>
<td>2</td>
<td>40</td>
<td>50</td>
</tr>
<tr>
<td>3</td>
<td>60</td>
<td>75</td>
</tr>
<tr>
<td>4</td>
<td>80</td>
<td>100</td>
</tr>
</tbody>
</table>

13. Zack completes 20 homework problems in 1 hour. Draw a graph of homework problems versus time. Find the value of the slope and interpret it in words.

14. The Jackson family rents 6 movies each month. Draw a graph of movies rented versus time. Find the value of the slope and interpret it in words.
15. Two weeks ago, Audrey earned $84 for 7 hours of work. This week, she earned $132 for 11 hours of work. Find the numerical value of the slope of the line that represents Audrey's earnings.

16. The ordered pairs (1, 4), (3, 12), and (5, 20) represent the distance $y$ that Jairo walks after $x$ seconds. Plot the ordered pairs on the coordinate plane and draw a line through the points.

Find the slope of the line and explain what the slope represents.

---

**Common Core Spiral Review**

Determine if each situation is proportional. Explain your reasoning. 7.RP.2

17. Taxi cab passengers are charged $2.50 upon entering a cab. They are then charged $1.00 for every mile traveled.

18. A restaurant charges $5 for one sandwich, $9.90 for two sandwiches, and $14.25 for three sandwiches.

19. | Tickets Purchased | 1 | 2 | 3 | 4 |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost ($)</td>
<td>7.50</td>
<td>15</td>
<td>22.50</td>
<td>30</td>
</tr>
</tbody>
</table>

20. | Cups of Flour | 3 | 6 | 9 | 12 |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Cups of Sugar</td>
<td>2</td>
<td>4</td>
<td>6</td>
<td>8</td>
</tr>
</tbody>
</table>

---

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**Real-World Link**

**Speed** The distance \( y \) a car travels after \( x \) hours can be represented by \( y = 65x \). The table and graph also represent the situation.

<table>
<thead>
<tr>
<th>Time (hours)</th>
<th>Distance (miles)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>130</td>
</tr>
<tr>
<td>3</td>
<td>195</td>
</tr>
<tr>
<td>4</td>
<td>260</td>
</tr>
</tbody>
</table>

![Graph showing distance vs. time]

1. Fill in the blanks to find the constant ratio.

\[
\text{distance traveled} \div \text{driving time} = \frac{130}{2} = \frac{195}{4}
\]

The constant ratio is \( \frac{195}{4} \) miles per hour.

2. The constant rate of change, or slope, of the line is \( \frac{\text{change in miles}}{\text{change in time}} \), which is equal to \( \frac{195 - 130}{3 - 2} \) or \( \frac{15}{1} \) miles per hour.

3. Write a sentence that compares the constant rate of change and the constant ratio.

Which **Mathematical Practices** did you use?

Shade the circle(s) that applies.

1. Persevere with Problems
2. Reason Abstractly
3. Construct an Argument
4. Model with Mathematics
5. Use Math Tools
6. Attend to Precision
7. Make Use of Structure
8. Use Repeated Reasoning
**Key Concept**

**Direct Variation**

**Words**
A linear relationship is a direct variation when the ratio of y to x is a constant, k. We say y varies directly with x.

**Symbols**
\[ \frac{y}{x} = k \text{ or } y = kx, \]
where \( k \neq 0 \)

**Example**

\[ y = 3x \]

When two variable quantities have a constant ratio, their relationship is called a direct variation. The constant ratio is called the constant of variation. The constant of variation is also known as the constant of proportionality.

In a direct variation equation, the constant rate of change, or slope, is assigned a special variable, \( k \).

**Example**

1. The height of the water as a pool is being filled is shown in the graph. Determine the rate in inches per minute.

Since the graph of the data forms a line, the rate of change is constant. Use the graph to find the constant of proportionality.

\[
\begin{align*}
\text{height} & \quad \text{time} \\
2 & \quad \frac{5}{1} \\
4 & \quad \frac{10}{1} \\
6 & \quad \frac{15}{1} \\
8 & \quad \frac{20}{1} \\
0.4 & \quad \frac{1}{4} \\
0.4 & \quad \frac{1}{4} \\
0.4 & \quad \frac{1}{4} \\
0.4 & \quad \frac{1}{4}
\end{align*}
\]

The pool fills at a rate of 0.4 inch every minute.

**Got it? Do this problem to find out.**

a. Two minutes after a diver enters the water, he has descended 52 feet. After 5 minutes, he has descended 130 feet. At what rate is the scuba diver descending?
Example

2. The equation \( y = 10x \) represents the amount of money \( y \) Julio earns for \( x \) hours of work. Identify the constant of proportionality. Explain what it represents in this situation.

\[
\begin{align*}
y &= kx \\
y &= 10x
\end{align*}
\]

Compare the equation to \( y = kx \), where \( k \) is the constant of proportionality.

The constant of proportionality is 10. So, Julio earns $10 for every hour that he works.

Got it? Do this problem to find out.

b. The distance \( y \) traveled in miles by the Chang family in \( x \) hours is represented by the equation \( y = 55x \). Identify the constant of proportionality. Then explain what it represents.

Determine Direct Variation

Not all situations with a constant rate of change are proportional relationships. Likewise, not all linear functions are direct variations.

Example

3. Pizzas cost $8 each plus a $3 delivery charge. Show the cost of 1, 2, 3, and 4 pizzas. Is there a direct variation?

<table>
<thead>
<tr>
<th>Number of Pizzas</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost ($)</td>
<td>11</td>
<td>19</td>
<td>27</td>
<td>35</td>
</tr>
</tbody>
</table>

- Cost \( \div \) number of pizzas \( = \frac{11}{1} \), \( \frac{19}{2} \) or 9.5,
- \( \frac{27}{3} \) or \( \frac{35}{4} \) or 8.75

There is no constant ratio and the line does not go through the origin. So, there is no direct variation.

Got it? Do this problem to find out.

c. Two pounds of cheese cost $8.40. Show the cost of 1, 2, 3, and 4 pounds of cheese. Is there a direct variation? Explain.
Example

4. Determine whether the linear relationship is a direct variation. If so, state the constant of proportionality.

<table>
<thead>
<tr>
<th>Time, ( x )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wages ($), ( y )</td>
<td>12</td>
<td>24</td>
<td>36</td>
<td>48</td>
</tr>
</tbody>
</table>

Compare the ratios to check for a common ratio.

\[
\frac{wages}{time} = \frac{12}{1} = \frac{24}{2} = \frac{36}{3} = \frac{48}{4} = \frac{12}{1}
\]

Since the ratios are the same, the relationship is a direct variation.
The constant of proportionality is \( \frac{12}{1} \).

Guided Practice

1. The number of cakes baked varies directly with the number of hours the caterers work. What is the ratio of cakes baked to hours worked? (Examples 1 and 2)

2. An airplane travels 780 miles in 4 hours. Make a table and graph to show the mileage for 2, 8, and 12 hours. Is there a direct variation? Explain. (Examples 3 and 4)

3. **Building on the Essential Question** How can you determine if a linear relationship is a direct variation from an equation? a table? a graph?

Rate Yourself!

How confident are you about direct variation? Check the box that applies.

---

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1. Veronica is mulching her front yard. The total weight of mulch varies directly with the number of bags of mulch.

   What is the rate of change?  \[ \text{Example 1} \]

2. The Spanish club held a car wash to raise money. The equation \( y = 5x \) represents the amount of money \( y \) club members made for washing \( x \) cars. Identify the constant of proportionality. Then explain what it represents in this situation.  \[ \text{Example 2} \]

3. A technician charges $25 per hour plus $50 for a house call to repair home computers. Make a table and a graph to show the cost for 1, 2, 3, and 4 hours of home computer repair service. Is there a direct variation?  \[ \text{Example 3} \]

<table>
<thead>
<tr>
<th>Time (h)</th>
<th>Charge ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>80</td>
</tr>
<tr>
<td>2</td>
<td>160</td>
</tr>
<tr>
<td>3</td>
<td>240</td>
</tr>
<tr>
<td>4</td>
<td>320</td>
</tr>
</tbody>
</table>

**Determine whether each linear relationship is a direct variation. If so, state the constant of proportionality.**  \[ \text{Example 4} \]

4. Pictures, \( x \) | 3 | 4 | 5 | 6
   Profit, \( y \) | 24 | 32 | 40 | 48

5. Minutes, \( x \) | 185 | 235 | 275 | 325
   Cost, \( y \) | 60 | 115 | 140 | 190

6. Year, \( x \) | 5 | 10 | 15 | 20
   Height, \( y \) | 12.5 | 25 | 37.5 | 50

7. Game, \( x \) | 2 | 3 | 4 | 5
   Points, \( y \) | 4 | 5 | 7 | 11
8. At a 33-foot depth underwater, the pressure is 29.55 pounds per square inch (psi). At a depth of 66 feet, the pressure reaches 44.4 psi. At what rate is the pressure increasing?

**Reason Abstractly** If \( y \) varies directly with \( x \), write an equation for the direct variation. Then find each value.

9. If \( y = 14 \) when \( x = 8 \), find \( y \) when \( x = 12 \).

10. Find \( y \) when \( x = 15 \) if \( y = 6 \) when \( x = 30 \).

11. If \( y = 6 \) when \( x = 24 \), what is the value of \( x \) when \( y = 7 \)?

12. Find \( x \) when \( y = 14 \), if \( y = 7 \) when \( x = 8 \).

**H.O.T. Problems** Higher Order Thinking

13. **Reason Inductively** Identify two additional values for \( x \) and \( y \) in a direct variation relationship where \( y = 11 \) when \( x = 18 \).

\[
\begin{align*}
x &= & \hspace{1cm} & y &= & \hspace{1cm} & \\
x &= & \hspace{1cm} & y &= & \hspace{1cm}
\end{align*}
\]

14. **Persevere with Problems** Find \( y \) when \( x = 14 \) if \( y \) varies directly with \( x^2 \), and \( y = 72 \) when \( x = 6 \).

15. **Model with Mathematics** Tom is drawing rectangles in which the length varies directly with the width. One of his rectangles has a width of 2 centimeters and a length of 3.6 centimeters. Draw and label a rectangle with a width of 3.5 centimeters that could be one of Tom’s rectangles. Then find the perimeter.
16. The money Shelley earns varies directly with the number of dogs she walks. How much does Shelley earn for each dog she walks?

Since the points on the graph lie in a straight line, the rate of change is a constant. The constant ratio is what Shelley earns per dog.

\[ \text{pay (\$)} \rightarrow \frac{2}{1}, \frac{4}{2}, \frac{6}{3}, \frac{8}{4}, \frac{10}{5} \]

Shelley earns $2.00 per dog.

17. A cake recipe requires \[3\frac{1}{4}\] cups of flour for 13 servings and \[4\frac{1}{2}\] cups of flour for 18 servings. How much flour is required to make a cake that serves 28?

Determine whether each linear relationship is a direct variation. If so, state the constant of variation.

18. | Age, \(x\) | 11 | 13 | 15 | 19 |
    | Grade, \(y\) | 5  | 7  | 9  | 11 |

19. | Price, \(x\) | 20 | 25 | 30 | 35 |
    | Tax, \(y\)  | 4  | 5  | 6  | 7  |

20. **Multiple Representations** Robert is in charge of the community swimming pool. Each spring he drains it in order to clean it. Then he refills the pool, which holds 120,000 gallons of water. Robert fills the pool at a rate of 10 gallons each minute.

a. **Words** What is the rate at which Robert will fill the pool? Is it constant?

b. **Graph** Graph the relationship on the grid shown.

c. **Algebra** Write an equation for the direct variation.
21. Determine if each equation represents a direct variation. Select yes or no.
   a. \( y = 4x + 1 \)  [ ] Yes  [ ] No
   b. \( y = 7.5x \)  [ ] Yes  [ ] No
   c. \( y = \frac{1}{15}x \)  [ ] Yes  [ ] No
   d. \( y = \frac{6}{x} \)  [ ] Yes  [ ] No

22. Place a checkmark in the column below the correct direct variation equation, if possible.

<table>
<thead>
<tr>
<th>Price, ( x )</th>
<th>20</th>
<th>30</th>
<th>40</th>
<th>50</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discount, ( y )</td>
<td>2</td>
<td>4</td>
<td>6</td>
<td>8</td>
</tr>
<tr>
<td>Seconds, ( x )</td>
<td>2</td>
<td>6</td>
<td>7</td>
<td>11</td>
</tr>
<tr>
<td>Feet, ( y )</td>
<td>30</td>
<td>90</td>
<td>105</td>
<td>165</td>
</tr>
<tr>
<td>Packages, ( x )</td>
<td>3</td>
<td>5</td>
<td>7</td>
<td>9</td>
</tr>
<tr>
<td>Crayons, ( y )</td>
<td>54</td>
<td>90</td>
<td>126</td>
<td>162</td>
</tr>
<tr>
<td>Hours, ( x )</td>
<td>1</td>
<td>4</td>
<td>7</td>
<td>10</td>
</tr>
<tr>
<td>Cost, ( y )</td>
<td>15</td>
<td>30</td>
<td>45</td>
<td>60</td>
</tr>
</tbody>
</table>

\( y = 18x \) [ ] \( y = 15x \) [ ] not a direct variation [ ]

---

**Common Core Spiral Review**

23. The table below shows the number of sheets of paper in various numbers of packages. Graph the data. \( \text{6.RP.3b} \)

<table>
<thead>
<tr>
<th>Number of Packages</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Sheets</td>
<td>50</td>
<td>100</td>
<td>150</td>
<td>200</td>
</tr>
</tbody>
</table>

24. The cost of various numbers of tickets to a festival is shown in the table. Graph the data. Then find the slope of the line. Explain what the slope represents. \( \text{6.RP.3b} \)

<table>
<thead>
<tr>
<th>Number of Tickets</th>
<th>5</th>
<th>10</th>
<th>20</th>
<th>25</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost ($)</td>
<td>40</td>
<td>80</td>
<td>160</td>
<td>200</td>
</tr>
</tbody>
</table>

---

Need more practice? Download more Extra Practice at connectED.mcgraw-hill.com.
Biomechanical Engineering

Did you know that more than 700 pounds of force are exerted on a 140-pound long-jumper during the landing? Biomechanical engineers understand how forces travel through the shoe to an athlete's foot and how the shoes can help reduce the impact of those forces on the legs. If you are curious about how engineering can be applied to the human body, a career in biomechanical engineering might be a great fit for you.
**Start Off on the Right Foot**

Use the information in the graph to solve each problem.

1. Find the constant rate of change for the data shown in the graph below Exercise 2. Interpret its meaning.

2. Is there a proportional relationship between the weight of an athlete and the forces that are generated from running? Explain your reasoning.

---

**Career Project**

It's time to update your career portfolio! Use the Internet or another source to research the fields of biomechanical engineering, biomedical engineering, and mechanical engineering. Write a brief summary comparing and contrasting the fields. Describe how they are all related.

---

What subject in school is the most important to you? How would you use that subject in this career?

---

90  Chapter 1  Ratios and Proportional Reasoning
Vocabulary Check

Complete each sentence using the vocabulary list at the beginning of the chapter. Then circle the word that completes the sentence in the word search.

1. A _________ is a ratio that compares two quantities with different kinds of units.
2. A rate that has a denominator of 1 unit is called a __________ rate.
3. A pair of numbers used to locate a point in the coordinate plane is an __________ pair.
4. (0, 0) represents the __________.
5. A _________ fraction has a fraction in the numerator, denominator, or both.
6. A _________ variation is the relationship between two variable quantities with a constant ratio.
7. The __________ is the rate of change between any two points on a line.
8. One of the four regions into which a coordinate plane is separated is called a __________.
9. A __________ is an equation stating that two ratios or rates are equal.
10. The rate of __________ describes how one quantity changes in relation to another.
11. __________ analysis is the process of including units of measurement when you compute.

Word Search:

I U M H P G N B W Z A F X O Q G H W M E M P
L Z E O X V D H B T U S A A U X E L P M O C
U Y K N N N U L S N H K D Z A J U R W X T I
G A B V X Y P C Y E L M E D C H A N G E U
G O J Y L C S T F G P Y J V R T R T Y F O V
A Q N N L W I L T M R R F J A E D E V O Y M
M N I P U O I C Z J M W O C N Z D X X C A A
A T G N N I E L B A M K B P T O T R G Z U F
R H I A E R Z G A T R U O X O S G M O N H M
U T R Z I A H R S A Y F Y Z F R L U E R D E
N C O D T G C F A O X O M W B T T O K W X W
I L S E M G D I M E N S I O N A L I P T Z N
Y H M R L C O I E Z R S A V L Z B P O E Y L
X S W C C W G J W U D G A I W I E Y C N O D
J U U V K O Z D H J K G W Z P U K M F W J
A G A L R B K Z X Q G H P L P M N B W T V
Use your Foldable to help review the chapter.

Get it?

Identify the Correct Choice  Write the correct term or number to complete each sentence.

1. When a rate is simplified so that it has a (numerator, denominator) of 1 unit, it is called a unit rate.

2. If Dinah can skate $\frac{1}{2}$ lap in 15 seconds, she can skate 1 lap in (7.5, 30) seconds.

3. Slope is the ratio of (horizontal change to vertical change, vertical change to horizontal change).

4. When two quantities have a constant ratio, their relationship is called a (direct, linear) variation.
Road Trip

The Jensen family took a trip in September. Sally is calculating the gas mileage in miles per gallon for her dad’s SUV. When he fills the tank, her dad records the miles on the SUV as 24,033. Her dad pays $83.58 to fill up the empty gas tank of the SUV.

Write your answers on another piece of paper. Show all of your work to receive full credit.

**Part A**
What is the size of the SUV’s tank in gallons? Round your answer to the nearest whole number.

**Part B**
At the destination, there was a quarter-tank of gas left in the vehicle and the miles were recorded as 24,297. Use a ratio equation to determine the gas mileage in miles per gallon for the trip. Round your answer to the nearest whole number.

**Part C**
Two months later, the Jensen family takes their mom’s sedan on a different trip. When she fills the tank, Sally’s mom records the miles on the sedan as 15,004. It takes $71.98 to fill up the vehicle’s empty gas tank. What is the size of the sedan’s tank in gallons? Round your answer to the nearest whole number.

**Part D**
At one point on the trip, the miles are recorded as 15,121 when the gas tank is 75% full. Use a ratio equation to determine the miles per gallon the sedan averages. Round your answer to the nearest whole number.

**Part E**
Which vehicle has the better gas mileage? Explain your reasoning.
Answering the Essential Question

Use what you learned about ratios and proportional reasoning to complete the graphic organizer.

**Essential Question**

HOW can you show that two objects are proportional...

... with a table?

... with a graph?

... with an equation?

**Answer the Essential Question.** HOW can you show that two objects are proportional?

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