Chapter 6
Equations and Inequalities

Essential Question
WHAT does it mean to say two quantities are equal?

Common Core State Standards
Content Standards
7.EE.3, 7.EE.4, 7.EE.4a, 7.EE.4b

Mathematical Practices
1, 2, 3, 4, 5, 7

Math in the Real World
Driving Suppose you live in a state where you must be at least 16 years of age to obtain a driver’s license. Circle the statement that represents this age.

Foldables Study Organizer
1 Cut out the Foldable on page FL5 of this book.
2 Place your Foldable on page 524.
3 Use the Foldable to help you learn about equations and inequalities.
**Vocabulary**

- Addition Property of Equality
- Addition Property of Inequality
- Coefficient
- Division Property of Equality
- Division Property of Inequality
- Equation
- Equivalent Equation
- Inequality
- Multiplication Property of Equality
- Multiplication Property of Inequality
- Solution
- Subtraction Property of Equality
- Subtraction Property of Inequality
- Two-step Equation
- Two-step Inequality

**Study Skill: Reading Math**

**Identify Key Information** Have you ever tried to solve a word problem and didn’t know where to start. Start by looking for key words in the text and images. Then write the important information in one sentence.

1. Highlight or circle key words in the following real-world problem.
   
   During a recent Super Bowl, millions of pounds of potato chips and tortilla chips were consumed. The number of pounds of potato chips consumed was 3.1 million pounds more than the number of pounds of tortilla chips. How many pounds of tortilla chips were consumed?

2. Write a sentence that summarizes the information provided. Include information from the text and the image.
**What Do You Already Know?**

Place a checkmark below the face that expresses how much you know about each concept. Then scan the chapter to find a definition or example of it.

- 😞 I have no clue.
- 😐 I've heard of it.
- 😊 I know it!

<table>
<thead>
<tr>
<th>Concept</th>
<th>😞</th>
<th>😐</th>
<th>😊</th>
<th>Definition or Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>inequalities</td>
<td></td>
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<tr>
<td>solving one-step equations</td>
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<td>solving inequalities by addition or subtraction</td>
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</table>

**When Will You Use This?**

Here are a few examples of how equations are used in the real world.

**Activity 1** Describe a situation when you only had a set amount of money to spend and you needed to buy a certain number of items. Then explain how you determined what you could buy.

**Activity 2** Go online at connectED.mcgraw-hill.com to read the graphic novel *Movie Night*. How much does each DVD cost? How much money do they need for popcorn?
Example 1
Write the phrase as an algebraic expression.
Phrase: five dollars more than Jennifer earned
Variable: Let \( d \) represent the number of dollars Jennifer earned.
Expression: \( d + 5 \)

Example 2
Is 3, 4, or 5 the solution of the equation \( x + 8 = 12 \)?

<table>
<thead>
<tr>
<th>Value of ( x )</th>
<th>( x + 8 = 12 )</th>
<th>Are both sides equal?</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>( 3 + 8 = 12 )</td>
<td>no</td>
</tr>
<tr>
<td></td>
<td>11 ( \neq 12 )</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>( 4 + 8 = 12 )</td>
<td>yes ( \checkmark )</td>
</tr>
<tr>
<td></td>
<td>12 = 12</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>( 5 + 8 = 12 )</td>
<td>no</td>
</tr>
<tr>
<td></td>
<td>13 ( \neq 12 )</td>
<td></td>
</tr>
</tbody>
</table>

The solution is 4 since replacing \( x \) with 4 results in a true sentence.

Quick Check
Words and Symbols: Write the phrase as an algebraic expression.
1. 3 more runs than the Pirates scored
2. a number decreased by eight
3. ten dollars more than Grace has

One-Step Equations: Identify the solution of each equation from the list given.
4. \( 8 + w = 17; 7, 8, 9 \)
5. \( d - 12 = 5; 16, 17, 18 \)
6. \( 6 = 3y; 2, 3, 4 \)
7. \( 7 \div c = 7; 0, 1, 2 \)
8. \( a + 8 = 23; 13, 14, 15 \)
9. \( 10 = 45 - n; 35, 36, 37 \)

How Did You Do?
Which problems did you answer correctly in the Quick Check? Shade those exercise numbers below.

1 2 3 4 5 6 7 8 9
HOW can bar diagrams or algebra tiles help you solve an equation?

In a recent year, 19 of the 50 states had a law banning the use of handheld cell phones while driving a school bus. Determine how many states did not have this law.

**Hands-On Activity 1**

You can represent this situation with an equation.

**Step 1**
The bar diagram represents the total number of states and the number of states that have passed a cell phone law. Fill in the missing information.

<table>
<thead>
<tr>
<th>states with a law</th>
<th>states that do not have a law</th>
</tr>
</thead>
<tbody>
<tr>
<td>-states-</td>
<td>--</td>
</tr>
</tbody>
</table>

**Step 2**
Write an equation from the bar diagram. Let $x$ represent the states that do not have a cell phone law for school bus drivers.

\[19 + x = 50\]

**Step 3**
Use the work backward strategy to solve the equation. Since

\[19 + x = 50, \; x = 50 - 19. \; \text{So,} \; x = \square.\]

Check \[19 + \square = 50 \checkmark\]

So, \square states did not have a law banning the use of cell phones by bus drivers.
Work with a partner to solve each problem.

1. Draw a bar diagram and write an addition equation to represent the following situation. Then solve the equation.

   The sum of a number and four is equal to 18.

   Equation: ____________  Solution: \( x = \) ____________

2. **Use Math Tools**  Jack collects postage stamps. He sold 7 of his stamps and had 29 stamps left. Complete the bar diagram below. Then write and solve a subtraction equation to find the number of stamps Jack had at the beginning.

   Equation: ____________  Solution: \( n = \) ____________

   So, Jack had ____________ stamps at the beginning.

3. Suppose Jack sold 15 stamps and had 21 stamps left. How would the bar diagram change?

   __________________________________________________________

   __________________________________________________________

4. **Reason Abstractly**  Suppose Jack had 40 stamps in the beginning and sold 7 of them. How would the bar diagram change? What equation could you write to represent the situation?

   __________________________________________________________

   __________________________________________________________
Hands-On Activity 2

Solve \( x - 3 = -2 \) using algebra tiles.

Remember a 1-tile and -1 tile combine to make a zero pair. You can add or subtract zero pairs from either side of an equation without changing its value.

**Step 1** Model the equation.

\[
\begin{array}{c}
x - 3 = -2 \\
\end{array}
\]

**Step 2** Add three 1-tiles to the left side of the mat and 1-tiles to the right side of the mat to form zero pairs on each side of the mat.

\[
\begin{array}{c}
x - 3 + 3 = -2 + 3 \\
\end{array}
\]

**Step 3** Remove all of the zero pairs from each side. There is 1-tile on the right side of the mat.

\[
\begin{array}{c}
x = 1 \\
\end{array}
\]

Therefore, \( x = \) _____________.

Check \( -3 = -2 \) ✔
MP Use Math Tools Work with a partner to solve each equation. Use algebra tiles. Show your work using drawings.

5. \( x + 4 = 4 \)  \( x = \) 

6. \( -2 = x + 1 \)  \( x = \) 

7. \( x - 1 = -3 \)  \( x = \) 

8. \( 4 = x - 2 \)  \( x = \) 

MP Construct an Argument Write a rule that you can use to solve addition equations without using models or a drawing.

HOW can bar diagrams or algebra tiles help you solve an equation?
An equation is a sentence stating that two quantities are equal. The value of a variable that makes an equation true is called the solution of the equation.

\[
x + 2 = 6 \\
-2 = -2 \\
x = 4
\]

The equations \(x + 2 = 6\) and \(x = 4\) are equivalent equations because they have the same solution, 4.

Circle the equations below that are equivalent to \(x = 3\). Use algebra tiles if needed.

\[
\begin{align*}
x + 3 &= 6 \\
x &= 1 = 6 \\
x + 3 &= 3 \\
x + 1 &= 4 \\
x + 2 &= 5
\end{align*}
\]

Video Games Robyn had some video games, and then she bought 4 more games. Now she has 10 games. This scenario can be described using the equation \(x + 4 = 10\).

1. What does \(x\) represent in the equation?

2. Write two different equations that are equivalent to \(x + 4 = 10\).
Subtraction Property of Equality

Words: The Subtraction Property of Equality states that the two sides of an equation remain equal when you subtract the same number from each side.

Symbols: If \( a = b \), then \( a - c = b - c \).

You can use bar diagrams and the work backward problem-solving strategy to solve equations arithmetically. Or, you can use the properties of equality to solve equations algebraically.

Examples

1. Solve \( x + 6 = 4 \). Check your solution.

   \[
   \begin{align*}
   x + 6 &= 4 & \text{Write the equation.} \\
   x &= 4 - 6 & \text{Subtraction Property of Equality} \\
   x &= -2 & \text{Simplify.}
   \end{align*}
   \]

   Check \( x + 6 = 4 \)

   \[
   \begin{align*}
   -2 + 6 &= 4 & \text{Write the original equation.} \\
   4 &= 4 & \text{Replace } x \text{ with } -2.
   \end{align*}
   \]

   So, the solution is \(-2\).

2. Solve \(-5 = b + 8\). Check your solution.

   \[
   \begin{align*}
   -5 &= b + 8 & \text{Write the equation.} \\
   -8 &= b & \text{Subtraction Property of Equality} \\
   -13 &= b & \text{Simplify.}
   \end{align*}
   \]

   Check \(-5 = b + 8\)

   \[
   \begin{align*}
   -5 &= -13 + 8 & \text{Replace } b \text{ with } -13. \\
   -5 &= -5 & \text{The sentence is true.}
   \end{align*}
   \]

   So, the solution is \(-13\).

Got it? Do these problems to find out.

Solve each equation. Check your solution.

a. \( y + 6 = 9 \)  \hspace{1cm} b. \( x + 3 = 1 \)  \hspace{1cm} c. \(-3 = a + 4\)
Example

3. An angelfish can grow to be 12 inches long. If an angelfish is 8.5 inches longer than a clown fish, how long is a clown fish?

Words: An angelfish is 8.5 inches longer than a clown fish.

Variable: Let \( c \) represent the length of the clown fish.

Equation: \[ 12 = c + 8.5 \]

Write the equation.

Subtraction Property of Equality

Simplify.

\[ 8.5 = 8.5 \]

\[ 3.5 = c \]

A clown fish is 3.5 inches long.

Got it? Do this problem to find out.

d. The highest recorded temperature in Warsaw, Missouri, is 118°F. This is 158° greater than the city's lowest recorded temperature. Find the lowest recorded temperature.

Addition Property of Equality

Words: The Addition Property of Equality states that the two sides of an equation remain equal when you add the same number to each side.

Symbols: If \( a = b \), then \( a + c = b + c \).

Example

4. Solve \( x - 2 = 1 \). Check your solution.

\[ x - 2 = 1 \] Write the equation.

\[ +2 = +2 \] Addition Property of Equality

\[ x = 3 \] Simplify.

The solution is 3. Check \( 3 - 2 = 1 \) √

Got it? Do these problems to find out.

e. \( y - 3 = 4 \) 
f. \( r - 4 = -2 \) 
g. \( q - 8 = -9 \)
Example

5. A pair of shoes costs $25. This is $14 less than the cost of a pair of jeans. Find the cost of the jeans.

Shoes are $14 less than jeans. Let \( j \) represent the cost of jeans.

\[
25 = j - 14
\]

Write the equation.

\[
+ 14 = + 14
\]

Addition Property of Equality

\[
39 = j
\]

Simplify.

The jeans cost $39.

Got it? Do this problem to find out.

h. The average lifespan of a tiger is 17 years. This is 3 years less than the average lifespan of a lion. Write and solve an equation to find the average lifespan of a lion.

Guided Practice

Solve each equation. Check your solution. (Examples 1, 2, and 4)

1. \( n + 6 = 8 \)

2. \( 7 = y + 2 \)

3. \( -7 = c - 6 \)

4. Orville and Wilbur Wright made the first airplane flights in 1903. Wilbur's flight was 364 feet. This was 120 feet longer than Orville's flight. Write an equation to represent the flights. Use a bar diagram if needed. Then solve to find the length of Orville's flight. (Examples 3 and 5)

5. Building on the Essential Question What are two methods for solving a real-world problem that can be represented by an equation?

Rate Yourself!

- I understand how to solve one-step addition and subtraction equations.

- I still have some questions about solving equations.

- No Problem! Go online to access a Personal Tutor.

440 Chapter 6 Equations and Inequalities
Solve each equation. Check your solution. (Examples 1, 2, and 4)

1. \(a + 3 = 10\)  
2. \(y + 5 = -11\)  
3. \(s - 8 = 9\)
4. \(5 = x + 8\)  
5. \(-2 = p - 1\)  
6. \(14 = s + 7\)

Use a bar diagram to solve arithmetically. Then use an equation to solve algebraically. (Examples 3 and 5)

7. Last week Tiffany practiced her bassoon a total of 7 hours. This was 2 hours more than she practiced the previous week. How many hours did Tiffany practice the previous week?

\[2 + p = 7\]

8. In a recent presidential election, Ohio had 18 electoral votes. This is 20 votes less than Texas had. How many electoral votes did Texas have?

\[t - 20 = 18\]

9. **Multiple Representations** Use the table to solve.

a. **Symbols** The difference in speeds of El Toro and T Express is 5 miles per hour. If El Toro has the greater speed, write and solve a subtraction equation to find its speed.

b. **Diagram** Voyage has a drop that is 22 feet less than El Toro. Draw a bar diagram to the right and write an equation to find the height of Voyage.

c. **Words** Let \(h\) represent the height of the Colossos roller coaster. Explain why \(h - 13 = 184\) and \(h - 34 = 163\) are equivalent equations. Then explain the meaning of the solution.
10. The sum of the measures of the angles of a triangle is \(180^\circ\). Write and solve an equation to find the missing measure.

11. The sum of the measures of a quadrilateral is \(360^\circ\). Write and solve an equation to find the missing measure.

**H.O.T. Problems** Higher Order Thinking

12. **Reason Inductively** Write an addition equation and a subtraction equation that have 10 as a solution.

13. **Find the Error** Aisha is finding \(b + 5 = -8\). Find her mistake and correct it.

\[
\frac{b + 5 = -8}{+5 + 5} = -3
\]

14. **Reason Abstractly** Suppose \(x + y = 11\) and the value of \(x\) increases by 2. If their sum remains the same, what must happen to the value of \(y\)? Justify your response

15. **Which One Doesn't Belong?** Identify the equation that does not belong with the other three. Explain your reasoning.

\[
\begin{align*}
x + 4 &= -2 \\
x + 5 &= -1 \\
x + 2 &= 8 \\
3 - x &= 1
\end{align*}
\]

16. **Reason Inductively** In the equation \(x + y = 5\), the value for \(x\) is a whole number greater than 2 but less than 6. Find the possible solutions for \(y\).
Extra Practice

Solve each equation. Check your solution.

17. \( r + 6 = -3 \)  
18. \( w - 7 = 11 \)  
19. \( k + 3 = -9 \)

\[
\begin{align*}
\text{Homework Help} & \\
0 + 6 &= -3 \\
-6 &= -6 \\
\hline
r &= -9
\end{align*}
\]

20. \( -1 = q - 8 \)
21. \( 9 = r + 2 \)
22. \( y + 15 = 11 \)

MP Use Math Tools Use a bar diagram to solve arithmetically. Then use an equation to solve algebraically.

23. The Miami Heat scored 79 points. This was 13 points less than the Chicago Bulls. How many points did the Chicago Bulls score?

24. Zach is \( 15 \frac{1}{2} \) years old. This is 3 years younger than his brother Lou. How old is Lou?

Copy and Solve Solve each equation. Check your solution. Show your work on a separate piece of paper.

25. The table shows a golfer's scores for four rounds of a recent U.S. Women's Open. Her total score was even with par. What was her score for the third round?

<table>
<thead>
<tr>
<th>Round</th>
<th>Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>First</td>
<td>-1</td>
</tr>
<tr>
<td>Second</td>
<td>-3</td>
</tr>
<tr>
<td>Third</td>
<td>5</td>
</tr>
<tr>
<td>Fourth</td>
<td>+2</td>
</tr>
</tbody>
</table>

26. \( a - 3.5 = 14.9 \)
27. \( b + 2.25 = 1 \)
28. \( -\frac{1}{3} = r - \frac{3}{4} \)

29. \( x - 2.8 = 9.5 \)
30. \( r - 8.5 = -2.1 \)
31. \( z - 9.4 = -3.6 \)

32. \( m + \frac{5}{6} = \frac{11}{12} \)
33. \( \frac{5}{6} + c = -\frac{11}{12} \)
34. \( s - \frac{1}{9} = \frac{5}{18} \)

Lesson 1 Solve One-Step Addition and Subtraction Equations 443
35. The model represents the equation \( x - 2 = 5 \). Determine if each statement is true or false.

   a. To solve the equation, add 2 positive counters to each side of the equation mat.  
      [ ] True  [ ] False

   b. To solve the equation, add 5 negative counters to each side of the equation mat.  
      [ ] True  [ ] False

   c. The value of \( x \) is 7.  
      [ ] True  [ ] False

36. Britney practiced the piano a total of 7 hours this week. This is 3 hours less than she practiced last week. Select the correct labels to complete the bar diagram that is used to find the number of hours \( w \) Britney practiced last week.

   How many hours did Britney practice the piano last week?

   [ ] 3 hours  [ ] 4 hours  [ ] 7 hours  [ ] 10 hours

---

37. \( 5(-4) = \)  

38. \( \frac{36}{9} = \)  

39. \( (-10)(-6) = \)  

40. \( \frac{-42}{-7} = \)  

41. \( (-3)(-2) = \)  

42. \( \frac{-54}{2} = \)  

43. While playing a round of golf, Tina had a score of three under par after the first three holes. Write and solve an equation to find Tina's average score per hole \( h \) after three holes.  

44. On Friday morning, the temperature dropped 2 degrees per hour for four hours. Write and solve an equation to find the total number of degrees \( d \) the temperature dropped on Friday morning.  

Need more practice? Download more Extra Practice at connectED.mcgraw-hill.com.
HOW do you know which operation to use when solving an equation?

Sakiya tutors students to earn money to buy a new Blu-ray player that costs $63. She is able to tutor seven hours in a week. How much should she charge per hour to have enough money by the end of the week?

What do you know?

What do you need to find?

Hands-On Activity

**Step 1** Draw a bar diagram that represents the money Sakiya needs to earn and the number of hours she is available to tutor that week.

**Step 2** Write an equation from the bar diagram. Let $x$ represent the amount she should charge each hour.

$$7x = 63$$

**Step 3** Use the work backward strategy to solve the equation. Since

$7x = 63$, $x = 63 \div 7$. So, $x = \square$.

Check $7 \times \square = 63 \checkmark$

So, Sakiya should charge $\square$ per hour.
Work with a partner to solve.

1. The screen on Lin's cell phone allows for 8 lines of text per message. The maximum number of characters for each message is 160. How many characters can each line hold? Complete the bar diagram below and write an equation. Then solve the equation.

---

**Analyze and Reflect**

Work with a partner to answer the following question.

2. **Make a Conjecture** Refer to Exercise 1. Suppose Lin's cell phone allows 4 lines of text and a maximum of 80 characters for each text message. How would the bar diagram and equation change?

---

**Create**

3. **Reason Abstractly** Keyani spent $70 for 4 hours of dance classes. How much did she spend per hour of dance class? Draw a bar diagram below and write an equation. Then solve the equation.

---

4. **Inquiry** How do you know which operation to use when solving an equation?
The expression $3x$ means $3$ times the value of $x$. The numerical factor of a multiplication expression like $3x$ is called a **coefficient**. So, $3$ is the coefficient of $x$.

The figure below illustrates the multiplication equation $3x = 6$.

Since there are $3$ $x$s, each $x$ is matched with $2$.

The solution of $3x = 6$ is $2$.

Write an equation that represents each of the models below. Identify the coefficient in your equation. Then solve.

1. 
   - **Equation:**
   - **Coefficient:**
   - **Solution:**

2. 
   - **Equation:**
   - **Coefficient:**
   - **Solution:**

Which **MP** Mathematical Practices did you use? Shade the circle(s) that applies.

1. Persevere with Problems
2. Reason Abstractly
3. Construct an Argument
4. Model with Mathematics
5. Use Math Tools
6. Attend to Precision
7. Make Use of Structure
8. Use Repeated Reasoning
**Division Property of Equality**

**Words**
The Division Property of Equality states that the two sides of an equation remain equal when you divide each side by the same nonzero number.

**Symbols**
If \( a = b \) and \( c \neq 0 \), then \( \frac{a}{c} = \frac{b}{c} \).

You can use the Division Property of Equality to solve multiplication equations.

**Examples**

1. **Solve** \( 20 = 4x \). **Check your solution.**
   
   \[
   \begin{align*}
   20 &= 4x & \text{Write the equation.} \\
   \frac{20}{4} &= \frac{4x}{4} & \text{Division Property of Equality} \\
   5 &= x & \text{Simplify.}
   \end{align*}
   \]

   **Check**
   
   \[
   \begin{align*}
   20 &= 4x & \text{Write the original equation.} \\
   20 &= 4(5) & \text{Replace } x \text{ with } 5. \\
   20 &= 20 & \text{This sentence is true.}
   \end{align*}
   \]

   So, the solution is 5.

2. **Solve** \( -8y = 24 \). **Check your solution.**
   
   \[
   \begin{align*}
   -8y &= 24 & \text{Write the equation.} \\
   \frac{-8y}{-8} &= \frac{24}{-8} & \text{Division Property of Equality} \\
   y &= -3 & \text{Simplify.}
   \end{align*}
   \]

   **Check**
   
   \[
   \begin{align*}
   -8y &= 24 & \text{Write the original equation.} \\
   -8(-3) &= 24 & \text{Replace } y \text{ with } -3. \\
   24 &= 24 & \text{This sentence is true.}
   \end{align*}
   \]

   So, the solution is \(-3\).

**Got it? Do these problems to find out.**

Solve each equation. **Check your solution.**

\[
\begin{align*}
\text{a. } 30 &= 6x \\
\text{b. } -6a &= 36 \\
\text{c. } -9d &= -72
\end{align*}
\]
Example

3. Leelah sent 574 text messages last week. On average, how many messages did she send each day?

Let \( m \) represent the number of messages Leelah sent.

\[
\frac{574}{7} = 7m
\]
Write the equation. There are 7 days in one week.

\[
\frac{574}{7} = 7m
\]
Division Property of Equality

\[
82 = m
\]
Simplify.

Leelah sent 82 messages on average each day.

Got it? Do this problem to find out.

d. Mrs. Acosta's car can travel an average of 24 miles on each gallon of gasoline. Write and solve an equation to find how many gallons of gasoline she will need for a trip of 348 miles.

Solve Arithmetically

You can use a bar diagram to solve an equation arithmetically.

\[
\begin{array}{c}
\text{text messages in 1 week: 574} \\
\hline
\text{7 messages in 1 day} \\
\text{Work backward to solve for } m \end{array}
\]

\[
m = \frac{574}{7} = 82
\]

Key Concept

Multiplication Property of Equality

Words

The Multiplication Property of Equality states that the two sides of an equation remain equal if you multiply each side by the same number.

Symbols

If \( a = b \), then \( ac = bc \).

You can use the Multiplication Property of Equality to solve division equations.

Example

4. Solve \( \frac{a}{-4} = -9 \).

\[
\frac{a}{-4} = -9
\]
Write the equation.

\[
\frac{a}{-4}(-4) = -9(-4)
\]
Multiplication Property of Equality

\[
a = 36
\]
Simplify.

Got it? Do these problems to find out.

e. \( \frac{y}{3} = -8 \)

f. \( \frac{m}{5} = -7 \)

g. \( \frac{30}{-6} = b \)
Example

5. The distance \( d \) Tina travels in her car while driving 60 miles per hour for 3 hours is given by the equation \( \frac{d}{3} = 60 \). How far did she travel?

\[ \frac{d}{3} = 60 \quad \text{Write the equation.} \]

\[ \frac{d}{3}(3) = 60(3) \quad \text{Multiplication Property of Equality} \]

\[ d = 180 \quad \text{Simplify.} \]

Tina traveled 180 miles.

Guided Practice

Solve each equation. Check your solution. (Examples 1, 2, and 4)

1. \( 6c = 18 \)

2. \( 24 = -8x \)

3. \( 7m = -28 \)

4. \( \frac{p}{9} = 9 \)

5. \( \frac{a}{12} = -3 \)

6. \( \frac{n}{10} = -4 \)

7. Antonia earns $6 per hour helping her grandmother. Write and solve an equation to find how many hours she needs to work to earn $48. (Example 3)

8. A shark can swim at an average speed of 25 miles per hour. At this rate, how far can a shark swim in 2.4 hours?

\[ \text{Use } r = \frac{d}{t}. \quad (\text{Example 5}) \]

9. Building on the Essential Question: How is the process for solving multiplication and division one-step equations like solving one-step addition and subtraction equations?

Rate Yourself!

How confident are you about solving one-step multiplication and division equations? Check the box that applies.

For more help, go online to access a Personal Tutor.
 Solve each equation. Check your solution. (Examples 1, 2, and 4)

1. \(7a = 49\) 
2. \(-6 = 2x\) 
3. \(-32 = -4b\) 
4. \(\frac{u}{6} = 9\) 
5. \(-8 = \frac{c}{-10}\) 
6. \(54 = -9d\)

7. \(-12y = 60\) 
8. \(\frac{r}{20} = -2\) 
9. \(\frac{a}{10} = -9\)

10. Brandy wants to buy a digital camera that costs $300. Suppose she saves $15 each week. In how many weeks will she have enough money for the camera? Use a bar diagram to solve arithmetically. Then use an equation to solve algebraically. (Example 3)

11. A race car can travel at a rate of 205 miles per hour. At this rate, how far would it travel in 3 hours? Use \(r = \frac{d}{t}\). Write an equation and then solve. (Example 5)

12. A certain hurricane travels at 20.88 kilometers per hour. The distance from Cuba to Key West is 145 kilometers. Write and solve a multiplication equation to find about how long it would take the hurricane to travel from Cuba to Key West.
13. **Multiple Representations** Kennedy saves $5.50 for each hour she works. She needs to save an additional $44 to buy an E-reader. How many more hours does Kennedy need to work to pay for the E-reader?

   a. **Diagram** Draw a bar diagram that represents the situation.

   ![Bar Diagram]

   b. **Algebra** Write an equation that represents the situation.

   c. **Words** Describe the process you would use to solve your equation. Then solve.

14. **Reason Abstractly** Describe a real-world situation in which you would use a division equation to solve a problem. Write your equation and then solve your problem.

   Situation:

   

   

   Equation: ___________________ Solution: ___________________

15. **Identify Structure** True or false. To solve the equation $5x = 20$ you can use the Multiplication Property of Equality. Explain your reasoning.

   

   

16. **Persevere with Problems** Solve $3|x| = 12$. Explain your reasoning.

   

   

17. **Persevere with Problems** Explain how you would solve $\frac{-30}{x} = 6$. Then solve the equation.

   

---

452 Chapter 6 Equations and Inequalities
Solve each equation. Check your solution.

18. \(-4j = 36\)
   \[-4j = 36\]
   \[\frac{-4j}{-4} = \frac{36}{-4}\]
   \[j = -9\]

19. \(-4s = -16\)

20. \(63 = -9d\)

21. \(\frac{m}{10} = 7\)
   \[\frac{m}{10} = 7\]
   \[m = 70\]

22. \(\frac{h}{3} = 12\)

23. \(\frac{g}{12} = -10\)

24. The width of a computer monitor is 1.25 times its height. Find the height of the computer monitor at the right. Use a bar diagram to solve arithmetically. Then use an equation to solve algebraically.

25. A dragonfly, the fastest insect, can fly a distance of 50 feet at a speed of 25 feet per second. Find the time in seconds. Write the equation in the form \(d = rt\), then solve.

26. **Find the Error** Raul is solving \(-6x = 72\). Find his mistake and correct it.
   \[-6x = 72\]
   \[\frac{-6x}{-6} = \frac{72}{-6}\]
   \[x = 12\]
27. The formula A = bh can be used to find the area A of a parallelogram with base b and height h. The parallelogram shown has an area of 56 square inches.

What is the length of the base?  

28. The table shows the prices of different satellite radio plans.
Mrs. Freedman paid $99 for m months of satellite radio under Plan A. Fill in each box to write a multiplication equation to represent the situation.

<table>
<thead>
<tr>
<th>Plan</th>
<th>Cost per Month ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>16.50</td>
</tr>
<tr>
<td>B</td>
<td>14.35</td>
</tr>
<tr>
<td>C</td>
<td>11.99</td>
</tr>
</tbody>
</table>

How many months of service did Mrs. Freedman purchase?

Common Core Spiral Review

Write each improper fraction as a mixed number and each mixed number as an improper fraction. 5.NF.3

29. \( \frac{10}{3} = \)  

30. \( \frac{40}{7} = \)  

31. \( \frac{101}{100} = \)  

32. \( 2\frac{2}{7} = \)  

33. \( 3\frac{1}{4} = \)  

34. \( 10\frac{5}{9} = \)  

Divide. 6.NS.3

35. \( 6 \div 1.5 = \)  

36. \( 3.6 \div 0.4 = \)  

37. \( 2.73 \div 1.3 = \)  

Multiply. Write in simplest form. 5.NF.4

38. \( \frac{2}{9} \times \frac{7}{5} = \)  

39. \( \frac{3}{4} \times 7 = \)  

40. \( \frac{5}{8} \times \frac{4}{15} = \)  

Need more practice? Download more Extra Practice at connectED.mcgraw-hill.com.
Inquiry Lab
Solve Equations with Rational Coefficients

**Inquiry** HOW can you use bar diagrams to solve equations with rational coefficients?

Two thirds of Chen's homeroom class plan to participate in the school talent show. If 16 students from the class plan to participate, how many students are in the homeroom class?

What do you know? 

What do you need to find? 

**Hands-On Activity**
You can represent the situation above with an equation.

**Step 1** Draw a bar diagram that represents the total number of students in the class and how many plan to participate.

--- number of students in class ---

---

number of students who plan to participate

**Step 2** Write an equation from the bar diagram. Let \( c \) represent the total number of students in the class.

**Step 3** Find the number of students represented by the sections of the bar. Write that number in each section of the bar in Step 1.

Since each section represents 8 students, there are \( 8 \times 3 \) or \( \boxed{24} \) students in the class.

**Check**

\[
\frac{2}{3} \times 24 = \frac{2}{3} \times \frac{24}{1} = \frac{48}{3} = 16 \checkmark
\]

connectED.mcgraw-hill.com
Investigate

Work with a partner to solve the following problem.

1. Eliana is spending $\frac{3}{5}$ of her monthly allowance on a costume for the talent show. She plans to spend $24. Draw a bar diagram to represent the situation. Then write and solve an equation to find the amount of Eliana's monthly allowance.

Equation: _____  Solution: _____

Analyze and Reflect

Work with a partner to answer the following question.

2. **Make a Conjecture** Suppose Eliana planned on spending $\frac{3}{4}$ of her monthly allowance on a costume. How would the diagram and equation be different?

Create

3. **Model with Mathematics** Write a real-world problem that could be represented by the equation $\frac{2}{3}x = 12$. Then solve the equation.

4. **Inquiry** How can you use bar diagrams to solve equations with rational coefficients?
Social Networks: Three-fourths of the students in Aaliyah's class belong to a social network. There are 15 students in her class that belong to a social network.

1. Create a bar diagram and shade \( \frac{3}{4} \) or 0.75, of it.

![Diagram](image)

Label 15 along the bottom to show the amount of the bar that represents 15 students.

2. Based on the diagram, circle the equation that can be used to find \( c \), the number of students in Aaliyah's class.

\[
15c = \frac{3}{4} \quad 0.75c = 15 \quad 4c = 15
\]

3. Based on what you know about solving equations, explain how you could solve the equation you circled in Exercise 2.

4. How many students are in Aaliyah's class?

Which Mathematical Practices did you use?
Shade the circle(s) that applies.

1. Persevere with Problems
2. Reason Abstractly
3. Construct an Argument
4. Model with Mathematics
5. Use Math Tools
6. Attend to Precision
7. Make Use of Structure
8. Use Repeated Reasoning
Decimal Coefficients

If the coefficient is a decimal, divide each side by the coefficient.

**Example**

1. Solve $16 = 0.25n$. Check your solution.

   
   \[
   16 = 0.25n \quad \text{Write the equation.}
   \]

   \[
   \frac{16}{0.25} = \frac{0.25n}{0.25} \quad \text{Division Property of Equality}
   \]

   \[
   64 = n \quad \text{Simplify.}
   \]

   **Check** $16 = 0.25n$  
   
   \[
   16 \div 0.25 \cdot 64 \quad \text{Write the original equation}
   \]

   \[
   16 = 16 \quad \text{Replace } n \text{ with } 64.
   \]

   The solution is $64$.

**Got it?** Do these problems to find out.

a. $6.4 = 0.8m$  
   
   b. $-2.8p = 4.2$  
   
   c. $-4.7k = -10.81$

**Example**

2. Jaya's coach agreed to buy ice cream for all of the team members. Ice cream cones are $2.40 each. Write and solve an equation to find how many cones the coach can buy with $30.

   Let $n$ represent the number of cones the coach can buy.

   \[
   2.4n = 30 \quad \text{Write the equation; } 2.40 = 2.4.
   \]

   \[
   \frac{2.4n}{2.4} = \frac{30}{2.4} \quad \text{Division Property of Equality}
   \]

   \[
   n = 12.5 \quad \text{Simplify.}
   \]

   Since the number of ice cream cones must be a whole number, there is enough money for 12 ice cream cones.

**Got it?** Do this problem to find out.

   d. Suppose the ice cream cones cost $2.80 each. How many ice cream cones could the coach buy with $42?

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Fraction Coefficients

Recall that two numbers with a product of 1 are called multiplicative inverses, or reciprocals. If the coefficient in a multiplication equation is a fraction, multiply each side by the reciprocal of the coefficient.

Examples

3. Solve \( \frac{3}{4}x = \frac{12}{20} \)

Write the equation.

\[
\begin{align*}
\left( \frac{4}{3} \right) \cdot \frac{3}{4}x &= \left( \frac{4}{3} \right) \cdot \frac{12}{20} \\
\frac{11}{3} \cdot \frac{3}{4}x &= \frac{11}{3} \cdot \frac{12}{20} \\
\frac{4}{5} &= \frac{4}{5}
\end{align*}
\]

Divide by common factors.

Simplify. Check the solution.

4. Solve \(-\frac{7}{9}d = 5\). Check your solution.

Write the equation.

\[
\begin{align*}
\left( \frac{-9}{7} \right) \cdot \left( -\frac{7}{9} \right) &= \left( \frac{-9}{7} \right) \cdot 5 \\
\left( \frac{-9}{7} \right) &= \left( \frac{-9}{7} \right) \cdot \frac{5}{1} \\
\frac{1}{1} &= \frac{-45}{7} \text{ or } \frac{-63}{7}
\end{align*}
\]

Divide by common factors.

Simplify.

Check \(-\frac{7}{9}d = 5\)

Write the original equation.

\[
\begin{align*}
\left( \frac{-9}{7} \right) &= \left( \frac{-9}{7} \right) \cdot \frac{5}{1} \\
\frac{-45}{7} &= \frac{-45}{7} \cdot 5 \\
\frac{315}{63} &= \frac{315}{63} \div 5 \\
5 &= 5 \checkmark
\end{align*}
\]

This sentence is true.

Got it? Do these problems to find out.

e. \( \frac{1}{2}x = 8 \)

f. \( -\frac{3}{4}x = 9 \)

g. \( \frac{7}{8}x = \frac{21}{64} \)
Example

5. Valerie needs $\frac{2}{3}$ yard of fabric to make each hat for the school play. Write and solve an equation to find how many hats she can make with 6 yards of fabric.

Write and solve a multiplication equation. Let $n$ represent the number of hats.

$$\frac{2}{3}n = 6$$  
Write the equation.

$$\left(\frac{3}{2}\right) \cdot \frac{2}{3}n = \left(\frac{3}{2}\right) \cdot 6$$  
Multiply each side by $\frac{3}{2}$.

$$n = 9$$  
Simplify.

So, Valerie can make 9 hats.

Guided Practice

Solve each equation. Check your solution. (Examples 1, 3, and 4)

1. $1.6k = 3.2$

2. $-2.5b = 20.5$

3. $\frac{-1}{2} = \frac{-5}{18}h$

Write and solve an equation. (Examples 2 and 5)

4. The average growth of human hair is 0.5 inch per month. Find how long it takes a human to grow 3 inches of hair.

Equation:  
Solution:

5. Three fourths of the fruit in a refrigerator are apples. There are 24 apples in the refrigerator. How many pieces of fruit are in the refrigerator?

Equation:  
Solution:

6. Building on the Essential Question  What is the process for solving a multiplication equation with a rational coefficient?

Rate Yourself!

Are you ready to move on? Shade the section that applies.

YES  ?  NO

For more help, go online to access a Personal Tutor.
Solve each equation. Check your solution. (Examples 1, 3, and 4)

1. \(1.2x = 6\)
2. \(14.4 = -2.4b\)
3. \(-3.6h = -10.8\)

4. \(\frac{2}{5}t = \frac{12}{25}\)
5. \(-3\frac{1}{3} = -\frac{1}{2}g\)
6. \(-\frac{7}{9}m = \frac{11}{6}\)

7. **Financial Literacy** Dillon deposited \(\frac{3}{4}\) of his paycheck into the bank. The deposit slip shows how much he deposited. Write and solve an equation to find the amount of his paycheck. (Example 2)

   Equation: \(\underline{\hspace{2cm}}\)  Solution: \(\underline{\hspace{2cm}}\)

8. Twenty-four students brought their permission slips to attend the class field trip to the local art museum. If this represented eight tenths of the class, how many students are in the class? Use a bar diagram to solve arithmetically. Then use an equation to solve algebraically. (Example 5)

   Equation: \(\underline{\hspace{2cm}}\)  Solution: \(\underline{\hspace{2cm}}\)

9. **Justify Conclusions** Seventy-five percent, or 15, of the students in Emily’s homeroom class are going on a field trip. Two thirds, or 12, of the students in Santiago’s homeroom class are going on the field trip. Which class has more students? Justify your answer.

   Equation: \(\underline{\hspace{2cm}}\)  Solution: \(\underline{\hspace{2cm}}\)
10. **Reason Abstractly** Nora and Ryan are making stuffed animals for a toy drive. The table shows the fabric purchases they made. Who purchased the more expensive fabric? Explain your reasoning.

<table>
<thead>
<tr>
<th>Purchaser</th>
<th>Amount Purchased (yd)</th>
<th>Amount Paid ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nora</td>
<td>( \frac{2}{3} )</td>
<td>4</td>
</tr>
<tr>
<td>Ryan</td>
<td>0.8</td>
<td>6</td>
</tr>
</tbody>
</table>

**H.O.T. Problems** Higher Order Thinking

11. **Reason Inductively** Complete the statement: If \( 8 = \frac{m}{4} \), then \( m - 12 = \) \[ \square \]. Explain.

12. **Which One Doesn't Belong?** Identify the pair of numbers that does not belong with the other three. Explain.

\[
\begin{align*}
\frac{9}{6}, \frac{6}{9} & \quad 4, \frac{1}{4} \\
\frac{3}{5}, 5 & \quad \frac{2}{7}, \frac{7}{2}
\end{align*}
\]

13. **Persevere with Problems** The formula for the area of a trapezoid is \( A = \frac{1}{2}h(b_1 + b_2) \), where \( b_1 \) and \( b_2 \) are both bases and \( h \) is the height. Find the value of \( h \) in terms of \( A, b_1, \) and \( b_2 \). Justify your answer.

14. **Model with Mathematics** Write a real-world problem that can be represented by the equation \( 224 = 3.5r \). Then solve the problem and explain the solution.
Extra Practice

Solve each equation. Check your solution.

15. \(0.4q = 2.8\)
   \[
   \frac{0.4q}{0.4} = \frac{2.8}{0.4} \\
   q = 7
   \]

16. \(-5w = -24.5\)

17. \(-22.8 = 6n\)

18. \(\frac{7}{8}k = \frac{5}{6}\)
   \[
   \left(\frac{5}{6}\right) \cdot \frac{7}{8}k = \frac{5}{6} \\
   \frac{7}{8}k = \frac{5}{6} \\
   k = \frac{40}{42} \div \frac{20}{21}
   \]

19. \(-6\frac{1}{4} = \frac{3}{5}c\)

20. \(-\frac{4}{7}v = -8\frac{2}{3}\)

21. The Mammoth Cave Discovery Tour includes an elevation change of 140 feet. This is \(\frac{7}{15}\) of the elevation change on the Wild Cave Tour. What is the elevation change on the Wild Cave Tour? Use a bar diagram to solve arithmetically. Then use an equation to solve algebraically.

Equation: \(\Box\)  Solution: \(\Box\)

22. **Model with Mathematics** Refer to the graphic novel frame below. Write and solve an equation to find how many movies they have time to show.

   Equation: \(\Box\)  Solution: \(\Box\)

   We decided that the movie night will last four hours. Each movie takes 175 hours.
   So how many movies can we show?
   We are planning a movie night for our school.
   And we want to make sure we leave a half-hour for eating popcorn!
23. Which of the following high speed trains are traveling at a rate of 150 miles per hour? Select all that apply.
   - [ ] A train that travels 100 miles in \(\frac{2}{3}\) hour
   - [ ] A train that travels 160 miles in \(\frac{5}{6}\) hour
   - [ ] A train that travels 125 miles in \(\frac{4}{5}\) hour
   - [ ] A train that travels 90 miles in \(\frac{3}{5}\) hour

24. The table shows the results of a survey. Of those surveyed, 275 students said they prefer pop music.

   Write an equation that could be used to find the total number of students who were surveyed.

   How many students were surveyed?

<table>
<thead>
<tr>
<th>Music Preference</th>
<th>Fraction of Students</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jazz</td>
<td>(\frac{1}{3})</td>
</tr>
<tr>
<td>Pop</td>
<td>(\frac{5}{8})</td>
</tr>
<tr>
<td>Rap</td>
<td>(\frac{1}{4})</td>
</tr>
</tbody>
</table>

Common Core Spiral Review

Use the order of operations to evaluate each expression. 6.EE.2c

25. \(6 \times 4 - 2 = \)     26. \(70 - 5 \times 4 = \)     27. \(18 \div 2 - 7 = \)

28. Write add, divide, multiply, and subtract in the correct order to complete the following sentence. 6.EE.2c

   When using the order of operations to evaluate an expression, always ________ and ________ before you ________ and ________.

29. Used paperback books are $0.25, and hardback books are $0.50. If you buy 3 paperback books and 5 hardback books, how much money do you spend?

   Expression: ___________________  Solution: ___________________

30. Suppose you order 2 pizzas, 2 garlic breads, and 1 order of BBQ wings. How much change would you receive from $30?

   Expression: ___________________  Solution: ___________________

<table>
<thead>
<tr>
<th>Item</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>14&quot; pizza</td>
<td>$8</td>
</tr>
<tr>
<td>garlic bread</td>
<td>$2</td>
</tr>
<tr>
<td>BBQ wings</td>
<td>$4</td>
</tr>
</tbody>
</table>

**Inquiry** HOW can a bar diagram or algebra tiles help you solve a real-world problem?

Latoya plays basketball and tennis. She has two basketballs and three tennis balls that weigh a total of 48 ounces. Each tennis ball weighs 2 ounces. What is the weight of a basketball?

**Hands-On Activity 1**

You can use a bar diagram to represent the situation.

**Step 1** Draw a bar diagram that represents the total weight.

```
<table>
<thead>
<tr>
<th>basketball</th>
<th>tennis</th>
<th>tennis</th>
</tr>
</thead>
<tbody>
<tr>
<td>?</td>
<td>?</td>
<td>2 oz</td>
</tr>
</tbody>
</table>
```

**Step 2** Write an equation that is modeled by the bar diagram. Let \( x \) represent the weight of a basketball.

\[
2x + 6 = 48
\]

**Step 3** Use the bar diagram to solve the equation. Subtract the weight of the tennis balls, \( \square \) ounces, from the total weight, \( \square \) ounces.

The two basketballs together weigh \( \square - \square \), or \( \square \) ounces.

Divide the weight by \( \square \) to find the weight of one basketball.

So, \( x = \square \). The weight of one basketball is \( \square \div \square \), or \( \square \) ounces.

Check \( 2 \cdot \square + 6 = 48 \)

The weight of one basketball is \( \square \) ounces.
**Hands-On Activity 2**

You can use algebra tiles to model and solve the equation \(4x - 2 = 10\).

**Step 1**  
Model the equation.

\[
\begin{array}{c}
4x - 2 = 10 \\
\end{array}
\]

**Step 2**  
Add \(\square\) 1-tiles to each side of the mat to form zero pairs on the left side.

\[
\begin{array}{c}
4x - 2 + 2 = 10 + 2 \\
\end{array}
\]

**Step 3**  
Remove both zero pairs from the left side so that the variable is by itself.

\[
\begin{array}{c}
4x = 12 \\
\end{array}
\]

**Step 4**  
Divide the remaining tiles into \(\square\) equal groups.

\[
\begin{array}{c}
\frac{4x}{4} = \frac{12}{4} \\
\end{array}
\]

So, \(x = \square\).

**Check**  
\(4 \cdot \square - 2 = 10\) \(\checkmark\)

466  Chapter 6  Equations and Inequalities
Work with a partner to solve the following problem.

1. **Reason Abstractly** Ryan is saving money to buy a skateboard that costs $85. He has already saved $40. He plans to save the same amount each week for three weeks. Draw a bar diagram. Then write an equation. How much should Ryan save each week?

2. \(2x + 1 = 5\) \(x = \)  
3. \(3x + 2 = 11\) \(x = \)

4. \(4x + 3 = -5\) \(x = \)
5. \(2x - 1 = 7\) \(x = \)

6. \(5x - 2 = -7\) \(x = \)
7. \(3x - 4 = 5\) \(x = \)
8. **Reason Inductively** Work with a partner. Read the steps to model and solve an equation using algebra tiles. Then circle each correct equation.

<table>
<thead>
<tr>
<th>Steps to Solve</th>
<th>Choices of Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Add three 1-tiles to each side of the mat.</td>
<td>$2x + 3 = 15$</td>
</tr>
<tr>
<td>• Divide tiles into two equal groups.</td>
<td>$3x + 2 = 15$</td>
</tr>
<tr>
<td>• Add four 1-tiles to each side of the mat.</td>
<td>$3x - 4 = 11$</td>
</tr>
<tr>
<td>• Divide tiles into three equal groups.</td>
<td>$3x + 4 = 11$</td>
</tr>
<tr>
<td>• Remove seven 1-tiles from each side of the mat.</td>
<td>$4x - 3 = 11$</td>
</tr>
<tr>
<td>• Divide tiles into three equal groups.</td>
<td></td>
</tr>
<tr>
<td>• Add two $-1$-tiles to each side of the mat.</td>
<td>$7x + 3 = 10$</td>
</tr>
<tr>
<td>• Remove two zero pairs from the left side of the mat.</td>
<td>$3x + 7 = 10$</td>
</tr>
<tr>
<td>• Divide tiles into five equal groups.</td>
<td>$3x - 7 = 10$</td>
</tr>
</tbody>
</table>

9. **Construct an Argument** What did you observe while choosing the correct equations in the table above?

10. **Model with Mathematics** Write a real-world problem and an equation that the bar diagram below could represent. Then solve your problem.

```
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>540</td>
</tr>
<tr>
<td>--------------------</td>
<td>-------------------</td>
</tr>
<tr>
<td></td>
<td>200</td>
</tr>
<tr>
<td>--------------------</td>
<td>-------------------</td>
</tr>
<tr>
<td></td>
<td>?</td>
</tr>
<tr>
<td>--------------------</td>
<td>-------------------</td>
</tr>
<tr>
<td></td>
<td>?</td>
</tr>
</tbody>
</table>
```

11. **Inquiry** HOW can a bar diagram or algebra tiles help you solve a real-world problem?


**Real-World Link**

**Balloons** A company charges $2 for each balloon in an arrangement and a $3 delivery fee. You have $9 to spend. The equation $2x + 3 = 9$, where $x$ is the number of balloons, represents the situation. Work backward to solve for $x$.

Start with the amount of money you have to spend. Subtract the $3 delivery fee. Since each balloon is $2, divide by two

So, you can purchase ___ balloons.

Check your work by substituting your solution into the equation.

$$2\left(\frac{\text{___}}{2}\right) + 3 = 9.$$  
$$\frac{1}{2} \cdot 3 = 9$$  
$$\frac{3}{2} = 9$$

1. How many balloons could you have purchased if there was a $1 delivery charge?

Start with the amount of money you have to spend. Subtract the $1 delivery fee. Since each balloon is $2, divide by two

Which **MP** Mathematical Practices did you use? Shade the circle(s) that applies.

1. Persevere with Problems
2. Reason Abstractly
3. Construct an Argument
4. Model with Mathematics
5. Use Math Tools
6. Attend to Precision
7. Make Use of Structure
8. Use Repeated Reasoning

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Solve Two-Step Equations

Recall that the order of operations ensures that numerical expressions, such as $2 \cdot 5 + 3$, have only one value. To reverse the operations, undo them in reverse order.

Start $\rightarrow 2 \rightarrow \text{times 5} \rightarrow \text{plus 3} \rightarrow \text{divided by 5} \leftrightarrow \text{minus 3} \leftrightarrow 13$

A **two-step equation**, such as $2x + 3 = 9$, has two different operations, multiplication and addition. To solve a two-step equation, undo the operations in reverse order of the order of operations.

**Step 1**  Undo the addition or subtraction first.

**Step 2**  Undo the multiplication or division.

**Examples**

1. Solve $2x + 3 = 9$. Check your solution.

   $2x + 3 = 9$  \hspace{1cm} Write the equation.

   $-3 = -3$  \hspace{1cm} Undo the addition first by subtracting 3 from each side.

   $2x = 6$  \hspace{1cm} Next, undo the multiplication by dividing each side by 2.

   $x = 3$  \hspace{1cm} Simplify.

   **Check** $2x + 3 = 9$  \hspace{1cm} Write the original equation.

   $2(3) + 3 \neq 9$  \hspace{1cm} Replace $x$ with 3.

   $9 = 9$  \hspace{1cm} The sentence is true.

   The solution is 3.
2. Solve \(3x + 2 = 23\). Check your solution.

\[
\begin{align*}
3x + 2 &= 23 & \text{Write the equation.} \\
-2 &= -2 & \text{Undo the addition first by subtracting 2 from each side.} \\
3x &= 21 \\
\frac{3x}{3} &= \frac{21}{3} & \text{Division Property of Equality} \\
x &= 7 & \text{Simplify.}
\end{align*}
\]

Check \(3x + 2 = 23\)

\[
\begin{align*}
3(7) + 2 &= 23 & \text{Replace x with 7.} \\
23 &= 23 & \text{The sentence is true.}
\end{align*}
\]

The solution is 7.

3. Solve \(-2y - 7 = 3\). Check your solution.

\[
\begin{align*}
-2y - 7 &= 3 & \text{Write the equation.} \\
+7 &= +7 & \text{Undo the subtraction first by adding 7 to each side.} \\
-2y &= 10 \\
\frac{-2y}{-2} &= \frac{10}{-2} & \text{Division Property of Equality} \\
y &= -5 & \text{Simplify.}
\end{align*}
\]

The solution is -5.

4. Solve \(4 + \frac{1}{5}r = -1\). Check your solution.

\[
\begin{align*}
4 + \frac{1}{5}r &= -1 & \text{Write the equation.} \\
-4 &= -4 & \text{Undo the addition first by subtracting 4 from each side.} \\
\frac{1}{5}r &= -5 \\
5 \cdot \frac{1}{5}r &= 5 \cdot (-5) & \text{Multiplication Property of Equality} \\
r &= -25 & \text{Simplify.}
\end{align*}
\]

The solution is -25.

Got it? Do these problems to find out.

Solve each equation. Check your solution.

\[
\begin{align*}
a. \ 2x + 4 &= 10 \\
b. \ 3x + 5 &= 14 \\
c. \ 5 &= 2 + 3x \\
d. \ 4x + 5 &= 13 \\
e. \ -5s + 8 &= -2 \\
f. \ -2 + \frac{2}{3}w &= 10
\end{align*}
\]
Example

5. Toya had her birthday party at the movies. It cost $27 for pizza and $8.50 per friend for the movie tickets. How many friends did Toya have at her party if she spent $78?

Solve Arithmetically

You can use a bar diagram to solve an equation arithmetically.

<table>
<thead>
<tr>
<th>pizza</th>
<th>$78</th>
</tr>
</thead>
<tbody>
<tr>
<td>tickets</td>
<td>$8.50n</td>
</tr>
</tbody>
</table>

$27 -$8.50n = $78

Subtract 27 from 78. Then divide by 8.5.
78 - 27 = 51; 51 ÷ 8.5 = 6

Let \( n \) represent the number of friends.

Equation: \[ 27 + 8.50n = 78 \]

Write the equation.

\[ -27 \]

Subtract 27 from each side.

\[ 8.50n = 51 \]

Division Property of Equality

\[ \frac{8.50n}{8.50} = \frac{51}{8.50} \]

Simplify.

\[ n = 6 \]

Toya can have 6 friends at her party.

Guided Practice

Solve each equation. Check your solution. (Examples 1–4)

1. \[ 13 = 1 + 4s \]
2. \[ -3y - 5 = 10 \]
3. \[ -7 = 1 + \frac{2}{3}n \]

4. Syreeta wants to buy some CDs that each cost $14, and a DVD that costs $23. She has $65. Write and solve an equation to find how many CDs she can buy. (Example 5)

Equation: 

Solution: 

5. **Building on the Essential Question** When solving an equation, explain why it is important to perform identical operations on each side of the equals sign.

Rate Yourself!

How well do you understand solving two-step equations? Circle the image that applies.

Clear Somewhat Clear Not So Clear

For more help, go online to access a Personal Tutor.

FOLDABLES Time to update your Foldable!
Independent Practice

Solve each equation. Check your solution. (Examples 1–4)

1. \(3x + 1 = 10\)
2. \(-3 + 8n = -5\)
3. \(4h - 6 = 22\)
4. \(-8s + 1 = 33\)
5. \(-4w - 4 = 8\)
6. \(5 + \frac{1}{7}b = -2\)

7. **Reason Abstractly** Cristiano is saving money to buy a bike that costs $189. He has saved $99 so far. He plans on saving $10 each week. In how many weeks will he have enough money to buy the bike? Use a bar diagram to solve arithmetically. Then use an equation to solve algebraically. (Example 5)

---

Solve each equation. Check your solution.

8. \(2r - 3.1 = 1.7\)
9. \(4t + 3.5 = 12.5\)
10. \(8m - 5.5 = 10.1\)

---

**Temperature** is usually measured on the Fahrenheit scale (°F) or the Celsius scale (°C). Use the formula \(F = 1.8C + 32\) to convert from one scale to the other.

a. Convert the temperature for Alaska’s record low in July to Celsius. Round to the nearest degree.

b. Hawaii’s record low temperature is \(-11°C\). Find the difference in degrees Fahrenheit between Hawaii’s record low temperature and the record low temperature for Alaska in January.

---

<table>
<thead>
<tr>
<th>Alaska Record Low Temperatures (°F) by Month</th>
</tr>
</thead>
<tbody>
<tr>
<td>January</td>
</tr>
<tr>
<td>April</td>
</tr>
<tr>
<td>July</td>
</tr>
<tr>
<td>October</td>
</tr>
</tbody>
</table>
12. **Model with Mathematics** Refer to the graphic novel frame below. Jamar figured that they will spend $39 for popcorn. Each movie cost $19. Write and solve an equation to find how many movies they can purchase.

---

**H.O.T. Problems** Higher Order Thinking

13. **Reason Inductively** Refer to Exercise 11. Is there a temperature in the table at which the number of degrees Celsius is the same as the number of degrees Fahrenheit? If so, find it. If not, explain why not.

14. **Persevere with Problems** Suppose your school is selling magazine subscriptions. Each subscription costs $20. The company pays the school half of the total sales in dollars. The school must also pay a one-time fee of $18. Write and solve an equation to determine the fewest number of subscriptions that can be sold to earn a profit of $200.

15. **Model with Mathematics** Write a real-world problem that can be represented by the equation $$\frac{(12 + 14) \times h}{2} = 52.$$ Then solve the problem.
Extra Practice

Solve each equation. Check your solution.

16. \(5x + 4 = 19\)
   \[\frac{5x + 4}{5} = \frac{19}{5}\]
   \[x = 3\]

17. \(6m + 1 = -23\)

18. \(5 + 4d = 37\)

19. \(-7y + 3 = -25\)

20. \(25 + \frac{11}{12}b = 47\)

21. \(15 - \frac{1}{2}b = -3\)

22. It costs \$7.50 to enter a petting zoo. Each cup of food to feed the animals is \$2.50. If you have \$12.50, how many cups can you buy? Use a bar diagram to solve arithmetically. Then use an equation to solve algebraically.

23. MP Multiple Representations  The perimeter of a rectangle is 48 centimeters. Its length is 16 centimeters. What is the width \(w\)?
   a. Draw a bar diagram that represents this situation.

   b. Write and solve an equation that represents this situation.

   c. How does solving the equation arithmetically compare to solving an equation algebraically?
24. Admission to an amusement park costs $15 and game tickets cost $0.50 each. Craig has $22 to pay for admission and game tickets. Select the correct labels to complete the bar diagram that can be used to find the number of game tickets t that Craig can purchase.

How many game tickets can Craig purchase?

25. A rental car company charges a fixed fee of $30 plus $0.05 per mile. Let c represent the total cost of renting a car and driving it m miles. Write an equation that could be used to find the total cost of renting a car and driving it any number of miles. The Boggs family paid $49.75 for their car rental. How many miles did they drive?

Common Core Spiral Review

Use the Distributive Property to rewrite each expression. 6.EE.3

26. \(2(x + 7) = \)  

27. \(G(10 + n) = \)  

28. \(5(k - 4) = \)

Factor each expression. 6.NS.1

29. \(5x + 5 \cdot 7 = \)  

30. \(4n + 4 \cdot 2 = \)

31. \(10t + 10 \cdot 3 = \)  

32. \(7v + 7 \cdot 8 = \)
**Inquiry**

HOW are equations in \( p(x + q) = r \) form different from \( px + q = r \) equations?

Mark has two summer jobs. He babysits and helps with the gardening. He works at each job three days a week and earns a total of $240. The table shows his earnings each day. How much does he earn each day babysitting?

<table>
<thead>
<tr>
<th>Job</th>
<th>Daily Earnings ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Babysitting</td>
<td>( x )</td>
</tr>
<tr>
<td>Gardening</td>
<td>30</td>
</tr>
</tbody>
</table>

What do you know?

What do you need to find?

**Hands-On Activity 1**

**Step 1** Draw a bar diagram that represents the situation.

![Bar diagram showing earnings]

**Step 2** Write an equation that is modeled by the bar diagram.

\[ 3(x + 30) = \quad \]

From the diagram, you can see that one third of Mark's total earnings is equal to \( x + 30 \). So, \( x + 30 = \frac{240}{3} \) or \( \phantom{0} \).

Mark earns \( \phantom{0} \) - $30, or \( \phantom{0} \) each day babysitting.
Vijay and his brother bought two hamburgers and two lemonades. The hamburgers cost $6 each. They spent a total of $16. How much did each lemonade cost?

**Hands-On Activity 2**

Use algebra tiles to model the situation described above.

**Step 1** Model $2(x + 6) = 16$ using algebra tiles. Use groups of $(x + 6)$ tiles.

![Algebra tiles model](image)

$2(x + 6) = 16$

**Step 2** Divide the tiles into equal groups on each side of the mat. Remove group from each side.

![Algebra tiles after division](image)

$x + 6 = 8$

**Step 3** Remove the same number of 1-tiles from each side.

![Algebra tiles after removal](image)

$x = 2$

So, $x = \_\_$ Each lemonade costs \_\_$. 

478 Chapter 6 Equations and Inequalities
Investigate

Work with a partner to model and solve each equation. Use a bar diagram for Exercises 1 and 2. Use algebra tiles for Exercises 3–6.

1. \(3(x + 5) = 21 \quad x = \quad \)

2. \(2(x - 3) = 10 \quad x = \quad \)

3. \(4(x + 1) = 8 \quad x = \quad \)

4. \(3(x + 2) = -12 \quad x = \quad \)

5. \(2(x - 1) = 6 \quad x = \quad \)

6. \(3(x - 4) = -3 \quad x = \quad \)
Work with a partner to write and solve an equation that represents each problem.

7. Refer to Activity 1. If Mark worked four days a week and made $360, how much did he earn babysitting each day?

8. Refer to Activity 2. If Vijay and his brother spent a total of $15, how much did each lemonade cost?

9. **Reason Inductively** After modeling an equation using algebra tiles, Angelina used the steps shown below to solve the equation. Write two different equations in \( p(x + q) = r \) form that Angelina could have solved.

   **Step 1**  
   Divide the tiles into three equal groups on both sides of the mat.

   **Step 2**  
   Remove two groups from each side.

   **Step 3**  
   Add four 1-tiles to each side.

   Equation 1:  
   Equation 2:

10. **Model with Mathematics** Write a real-world problem that can be represented by the equation \( 4(x + 15) = 140 \). Then solve the problem.

11. **Inquiry** How are equations in \( p(x + q) = r \) form different from \( px + q = r \) equations?
**Real-World Link**

**Museums** A new exhibit about dinosaurs is being constructed. The exhibit is a rectangle that is 36 feet long. It has a perimeter of 114 feet. Follow the steps to write an equation that can be used to find the width of the museum exhibit.

**Step 1** Draw a diagram to help visualize the exhibit. Label the length and width. Let \( w \) represent the width.

\[
\text{Length} = 36 \text{ ft} \]
\[
\text{Width} = w
\]

**Step 2** Write an expression that represents the sum of the length and width of the exhibit.

\[36 + w\]

**Step 3** Write an expression that represents twice the sum of the length and width.

\[2(36 + w)\]

**Step 4** Write an equation that represents the perimeter of the exhibit.

\[2(36 + w) = 114\]

---

**Which **Mathematical Practices** did you use? Shade the circle(s) that applies.**

1. Persevere with Problems
2. Reason Abstractly
3. Construct an Argument
4. Model with Mathematics
5. Use Math Tools
6. Attend to Precision
7. Make Use of Structure
8. Use Repeated Reasoning
Solve Two-Step Equations

An equation like \(2(w + 36) = 114\) is in the form \(p(x + q) = r\). It contains two factors, \(p\) and \((x + q)\), and is considered a two-step equation. Solve these equations using the properties of equality.

Examples

1. Solve \(3(x + 5) = 45\).

   Method 1  Solve arithmetically.

   \[
   \begin{array}{c|c|c|c}
   \hline
   \text{ } & \text{ } & \text{ } & \text{ } \\
   \hline
   x + 5 & x + 5 & x + 5 \\
   \hline
   \hline
   \text{ } & \text{ } & \text{ } & \text{ } \\
   \hline
   \text{ } & \text{ } & \text{ } & \text{ } \\
   \hline
   \text{ } & \text{ } & \text{ } & \text{ } \\
   \hline
   \text{ } & \text{ } & \text{ } & \text{ } \\
   \hline
   \end{array}
   \]

   Draw a bar diagram. From the diagram, you can see that \(x + 5 = 45 ÷ 3\) or 15. So, \(x = 15 - 5\) or 10.

   Method 2  Solve algebraically.

   \[
   \begin{align*}
   3(x + 5) &= 45 \\
   \frac{3(x + 5)}{3} &= \frac{45}{3} \\
   x + 5 &= 15 \\
   -5 &= -5 \\
   x &= 10
   \end{align*}
   \]

2. Solve \(5(n - 2) = -30\).

   \[
   \begin{align*}
   5(n - 2) &= -30 \\
   \frac{5(n - 2)}{5} &= \frac{-30}{5} \\
   n - 2 &= -6 \\
   +2 &= +2 \\
   n &= -4
   \end{align*}
   \]

Check Your Work

Remember to plug your solution back into the original equation to see if it makes a true statement.

a. ____________  
   b. ____________  
   c. ____________

Got it? Do these problems to find out.

a. \(2(x + 4) = 20\)  
   b. \(3(b - 6) = 12\)  
   c. \(-7(6 + c) = 49\)
Equations with Rational Coefficients

Sometimes the factor \(p\), in \(p(x + q)\), will be a fraction or decimal.

**Examples**

3. Solve \(\frac{2}{3}(n + 6) = 10\). Check your solution.

\[
\begin{align*}
\frac{2}{3}(n + 6) &= 10 & \text{Write the equation.} \\
\frac{3}{2} \cdot \frac{2}{3}(n + 6) &= \frac{3}{2} \cdot 10 & \text{Multiplication Property of Equality} \\
(n + 6) &= \frac{3}{2} \cdot \frac{10}{1} \\
 n + 6 &= \frac{3 \cdot 10}{2} = \frac{15}{1} & \text{Simplify.} \\
 \underline{-6} &= \underline{-6} & \text{Subtraction Property of Equality} \\
 n &= 9 & \text{Simplify.}
\end{align*}
\]

Check \(\frac{2}{3}(n + 6) = 10\). Write the original equation.

\[
\begin{align*}
\frac{2}{3}(9 + 6) &= 10 & \text{Replace } n \text{ with } 9. \text{ Is this sentence true?} \\
10 &= 10 & \checkmark \quad \text{The sentence is true.}
\end{align*}
\]

4. Solve \(0.2(c - 3) = -10\). Check your solution.

\[
\begin{align*}
0.2(c - 3) &= -10 & \text{Write the equation.} \\
\frac{0.2(c - 3)}{0.2} &= \frac{-10}{0.2} & \text{Division Property of Equality} \\
c - 3 &= -50 & \text{Simplify.} \\
\underline{+3} &= \underline{+3} & \text{Addition Property of Equality} \\
c &= -47 & \text{Simplify.}
\end{align*}
\]

Check \(0.2(c - 3) = -10\). Write the original equation.

\[
\begin{align*}
0.2(-47 - 3) &= -10 & \text{Replace } c \text{ with } -47. \text{ Is this sentence true?} \\
-10 &= -10 & \checkmark \quad \text{The sentence is true.}
\end{align*}
\]

**Got it?** Do these problems to find out.

\[
\begin{align*}
d. & \quad \frac{1}{4}(d - 3) = -15 \\
e. & \quad 0.75(6 + d) = 12 \\
f. & \quad (t + 3)\frac{5}{9} = 40
\end{align*}
\]
5. Jamal and two cousins received the same amount of money to go to a movie. Each boy spent $15. Afterward, the boys had $30 altogether. Write and solve an equation to find the amount of money each boy received.

Let $m$ represent the amount of money each boy received.

\[
\begin{align*}
3(m - 15) &= 30 \\
\frac{3(m - 15)}{3} &= \frac{30}{3} \\
\cancel{3}(m - 15) &= \cancel{3} \\
\frac{m - 15}{1} &= \frac{10}{1} \\
\cancel{1}(m - 15) &= \cancel{1}10 \\
\frac{15}{1} &= \frac{+15}{+15} \\
\frac{m}{1} - \frac{15}{1} &= \frac{25}{1} \\
\frac{m}{1} &= \frac{25}{1} \\
\frac{m}{1} &= 25 \\
m &= 25
\end{align*}
\]

So, each boy received $25.

Guided Practice

Solve each equation. Check your solution. (Examples 1–4)

1. $2(p + 7) = 18$          2. $(4 + g)(-11) = 121$
3. $(v + 5)(-\frac{1}{9}) = 6$          4. $0.8(m - 5) = 10$

5. Mr. Singh had three sheets of stickers. He gave 20 stickers from each sheet to his students and has 12 total stickers left. Write and solve an equation to find how many stickers were originally on each sheet. (Example 5)

Equation: ____________________________________
Solution: ____________________________________

6. ★ Building on the Essential Question What is the difference between $px + q = r$ and $p(x + q) = r$?

__________________________
Independent Practice

Solve each equation. Check your solution. (Examples 1-4)

1. \( 8(s + 3) = 72 \)

2. \( -7(z - 6) = -70 \)

3. \( \frac{1}{11}(t + 8)(-2) = 12 \)

4. \( \frac{8}{11}(n - 10) = 64 \)

5. \( -0.6(r + 0.2) = 1.8 \)

6. \( (w - \frac{4}{9})(-\frac{7}{3}) = -\frac{4}{5} \)

7. The length of each side of an equilateral triangle is increased by 5 inches, so the perimeter is now 60 inches. Write and solve an equation to find the original length of each side of the equilateral triangle. (Example 5)

   Equation: ___________________________  Solution: ___________________________

8. **Multiple Representations** Miguel and three of his friends went to the movies. They originally had a total of $40. Each boy had the same amount of money and spent $7.50 on a ticket. How much money did each boy have left after buying his ticket?

   a. **Model** Draw a bar diagram that represents the situation.

   [Diagram]

   b. **Algebra** Write and solve an equation that represents the situation.

   [Equation]

   c. **Words** Explain how you solved your equation.

   [Explanation]

   d. **Words** Compare the arithmetic solution and the algebraic solution.

   [Comparison]
9. Mrs. Sorenstam bought one ruler, one compass, and one mechanical pencil at the prices shown in the table for each of her 12 students.

<table>
<thead>
<tr>
<th>Item</th>
<th>Price ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>compass</td>
<td>1.49</td>
</tr>
<tr>
<td>mechanical pencil</td>
<td>0.59</td>
</tr>
<tr>
<td>ruler</td>
<td>0.49</td>
</tr>
</tbody>
</table>

a. Suppose Mrs. Sorenstam had 36 cents left after buying the school supplies. Write an equation to find the amount of money Mrs. Sorenstam initially had to spend on each student.

b. Describe a two-step process you could use to solve your equation. Then solve the equation.

---

**H.O.T. Problems** Higher Order Thinking

10. **Model with Mathematics** Write a real-world situation that can be represented by the equation \(2(n + 20) = 110\).

11. **Find the Error** Marisol is solving the equation \(6(x + 3) = 21\). Find her mistake and correct it.

\[
\begin{align*}
6(x + 3) &= 21 \\
-3 &= -3 \\
x &= \frac{18}{6} \\
&= 3
\end{align*}
\]

12. **Persevere with Problems** Solve \(p(x + a) = r\) for \(x\).

13. **Use Math Tools** Write an equation to represent the bar diagram at the right. Then write a real-world problem that can be represented by the equation and the diagram.
Extra Practice

Solve each equation. Check your solution.

14. \(0.25(3 + a) = 0.5\)  \[0.25(3 + a) = 0.5\]
\[\frac{0.25(3 + a)}{0.25} = \frac{0.5}{0.25}\]
\[3 + a = 2\]
\[a = -1\]

15. \(12(x - 20) = -48\)

16. \(-28 = 7(n + 3)\)

17. \((t + 9)20 = 140\)

18. \(\frac{5}{3}(6 + c) = -20\)

19. \((d - 3)\frac{2}{3} = 30\)

20. **Reason Abstractly** Anne bought a necklace for each of her three sisters. She paid $7 for each necklace. Suppose she had $9 left. Write and solve an equation to find how much money Anne had initially to spend on each sister.

   Equation: 

   Solution: 

Solve each equation. Check your solution.

21. \(\frac{3}{5}(t - 6) = -0.4\)

22. \((x + 5\frac{1}{2})0.75 = \frac{5}{8}\)

23. Mr. Gomez bought fruit to make fruit salad. He bought \(2\frac{1}{2}\) pounds of apples and spent $4.50 on apples and oranges. Write and solve an equation to determine the number of pounds of oranges Mr. Gomez bought.

<table>
<thead>
<tr>
<th>Fruit</th>
<th>Price per Pound ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>apples</td>
<td>1.20</td>
</tr>
<tr>
<td>bananas</td>
<td>0.50</td>
</tr>
<tr>
<td>grapes</td>
<td>1.50</td>
</tr>
<tr>
<td>oranges</td>
<td>1.20</td>
</tr>
</tbody>
</table>
24. A rectangular classroom is 32 feet long and has a perimeter of 120 feet. Label the drawing with the correct values to represent the situation. Let \( w \) represent the width of the classroom.

Write an expression that represents the sum of the length and width. 

Write an expression that represents twice the sum of the length and width. 

Write an equation you could use to find the perimeter of the classroom. 

What is the width of the classroom? 

25. Which of the following are operations that you should use to solve the equation \( p(x - q) = r \) for \( x \)? Select all that apply.

- Subtract \( q \) from both sides.
- Divide both sides by \( p \).
- Multiply both sides by \( p \).
- Add \( q \) to both sides.

---

**Common Core Spiral Review**

Solve each equation. 6.EE.6

26. \( x + 3 = 5 \) 
27. \( x - 2 = -6 \) 
28. \( 4x = 12 \)

29. \( -6x = -24 \) 
30. \( \frac{x}{2} = -1 \) 
31. \( \frac{x}{-3} = 1 \)

Write the number or numbers from the set \( \{-3, -2, -1, 0, 1, 2, 3\} \) that make each statement true. 6.EE.5

32. \( 4m = 12 \)  
33. \( y - 1 = 1 \)

34. \( v > 0 \)  
35. \( r \leq 0 \)

---

488 **Need more practice?** Download more Extra Practice at connectED.mcgraw-hill.com.
Case #1  Yard Work
Mike earned extra money by doing yard work for his neighbor. Then he spent $5.50 at the convenience store and four times that amount at the bookstore. Now he has $7.75 left.
How much money did Mike have before he went to the convenience store and the bookstore?

Understand  What are the facts?
You know Mike has $7.75 left. You need to find the amount before his purchases.

Plan  What is your strategy to solve this problem?
Start with the end result and work backward.

Solve  How can you apply the strategy?
He has $7.75 left.
Undo the four times $5.50 spent at the bookstore. $7.75
Since $5.50 \times 4$ is $22$, add $7.75$ and $22$. $+ \$22.00$
Undo the $5.50$ spent at the convenience store. $+ \$5.50$
Add $5.50$ and $29.75$. $\underline{\$35.25}$
So, Mike's starting amount was $35.25$.

Check  Does the answer make sense?
Assume Mike started with $35.25$. He spent $5.50$ and $22$. He had $35 - 5.50 - 22$ or $\underline{\$31.75}$ left. So, $35.25$ is correct.

Analyze the Strategy

MP  Construct an Argument  Describe how to solve a problem by working backward.
Case #2 Money
Marisa spent $8 on a movie ticket. Then she spent $5 on popcorn and one half of what was left on a drink. She had $2 left.

How much did she have initially?

1. Understand
Read the problem. What are you being asked to find?

I need to find

Underline key words and values. What information do you know?

I know Marisa has ___ left and that she spent ___, ___, and ___.

Is there any information that you do not need to know?

I do not need to know

2. Plan
Choose a problem-solving strategy.

I will use the ___ strategy.

3. Solve
Use your problem-solving strategy to solve the problem.
Marisa has $2 left.

Undo the half-of-what-was-left amount. Multiply by 2.

Undo the spent $5. Add $5.

Undo the spent $8. Add $8.

So, Marisa had ___ initially.

4. Check
Use information from the problem to check your answer.

Marisa’s initial amount:

Amount after spending $8:

Amount after spending $5:

Amount after spending half of what was left:
Work with a small group to solve the following cases. Show your work on a separate piece of paper.

**Case #3 Waterfalls**
Angel Falls in Venezuela is 3,212 feet high. It is 29 yards higher than 2.5 times the architectural height of the Empire State Building. Find the architectural height, in feet, of the Empire State Building.

**Case #4 Number Theory**
Travis works at a kite factory. He checks all the kites before they are packaged. Travis discovered that for every 28 kites that passed inspection, there were 7 kites that did not pass: 4 kites did not have tails, and 3 kites had the wrong colors.

Of the 476 kites Travis examined, how many did not have tails and how many had the wrong colors?

**Case #5 Time**
Timothy’s morning schedule is shown.
At what time does Timothy wake up if he arrives at school at 7:35 A.M.?

<table>
<thead>
<tr>
<th>Timothy’s Schedule</th>
</tr>
</thead>
<tbody>
<tr>
<td>Activity</td>
</tr>
<tr>
<td>Wake up</td>
</tr>
<tr>
<td>Get ready for school – ( \frac{3}{4} ) h</td>
</tr>
<tr>
<td>Walk to school – ( \frac{5}{12} ) h</td>
</tr>
</tbody>
</table>

**Case #6 Money**
Antonio has saved $28 to spend at the arcade. If he has 5 bills, how many of each kind of bill does he have?
Vocabulary Check

1. Define *equation*. Give an example of two equivalent equations.  *(Lesson 1)*

2. Fill in the blank with the correct term.  *(Lesson 2)*

   A ________ is the numerical factor of a multiplication expression like $3x$.

Skills Check and Problem Solving

Solve each equation. Check your solution.  *(Lessons 1–5)*

3. $21 + m = 33$

4. $a - 5 = -12$

5. $5f = -75$

6. $15 = \frac{b}{15}$

7. $19 = 4p + 5$

8. $3(n - 7) = -30$

9. Cameron has 11 adult Fantail goldfish. This is 7 fewer Fantail goldfish than his friend Julia has. Write and solve a subtraction equation to determine the number of Fantail goldfish $g$ that Julia has.  *(Lesson 1)*

   Equation: ____________________  Solution: ____________________

10. **MP Persevere with Problems** The pentagon shown is a regular pentagon, so each side has the same length. The perimeter of the pentagon is 22.5 centimeters. What is the value of $x$?

   $(x - 1.5)\text{ cm}$
Inquiry Lab
Solve Inequalities

**Inquiry** HOW is an inequality like an equation? How is it different?

Mr. Numkena volunteered to drive Hinto and his friends to the school dance. The car can carry up to 5 people, including the driver. How many friends can ride in the car with Hinto?

What do you know? ____________________________________________

What do you need to find? ________________________________________

**Hands-On Activity 1**

The real-world situation described above can be represented by the inequality $x + 2 \leq 5$. Let $x$ represent the friends that can ride with Hinto.

$\text{Mr. Numkena, Hinto, and friends} \quad \text{Maximum number of people}$

You can use a balance to model and solve the inequality $x + 2 \leq 5$.

**Step 1** On one side of a balance, place a paper bag and □ cubes to model $x + 2$.

**Step 2** On the other side of a balance, place □ cubes.

Add one cube to the bag at a time. Then complete the table.

<table>
<thead>
<tr>
<th>Number of Friends, $x$</th>
<th>$x + 2$</th>
<th>Less than or equal to 5?</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
<td>yes</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

So, up to □ friends can ride with Hinto to the school dance.

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An *inequality* is a mathematical sentence that compares quantities. The table shows two examples of inequalities.

<table>
<thead>
<tr>
<th>Words</th>
<th>Symbols</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x$ is less than two</td>
<td>$x &lt; 2$</td>
</tr>
<tr>
<td>$x$ is greater than or equal to four</td>
<td>$x \geq 4$</td>
</tr>
</tbody>
</table>

To solve an inequality means to find values for the variable that make the sentence true. You can use bar diagrams to solve inequalities.

**Hands-On Activity 2**

An airline charges for checked luggage that weighs more than 50 pounds. Mia’s suitcase currently weighs 35 pounds and she still needs to pack her shoes. Find the maximum amount her shoes can weigh so Mia will not be charged a fee.

**Step 1** In the bar diagram, write the maximum weight Mia’s luggage can be without a fee. Label the weight of Mia’s luggage without her shoes.

![Bar diagram showing weight of luggage without shoes.](image)

**Step 2** In the bar diagram, write an $x$ beside the bar that represents the weight of Mia’s luggage.

![Bar diagram showing weight of luggage with shoes.](image)

The weight of Mia’s suitcase plus the weight of her shoes must be less than or equal to the maximum luggage weight. This can be written as $35 + x \leq 50$.

Using the bar diagram, Mia’s shoes cannot weigh more than $50 - 35$ or ___ pounds.
Work with a partner to solve the following problems.

**Reason Inductively** For Exercises 1–3, assume the paper bag is weightless. Write the inequality represented by each balance. Then write the different possible numbers of cubes in the paper bag if the sides of each balance remain unlevel.

1. 
   - Inequality: 
   - Number of Cubes: 

2. 
   - Inequality: 
   - Number of Cubes: 

3. 
   - Inequality: 
   - Number of Cubes: 

**Reason Abstractly** At an amusement park, roller coaster riders are required to be at least 48 inches tall. Last year, Myron was 42 inches tall. Complete the bar diagram to determine the number of inches Myron needed to grow this year to be able to ride the roller coaster. Then write an inequality to represent the situation.

   - So, Myron needed to grow at least _______ inches.
   - Inequality: 

---

Inquiry Lab  Solve Inequalities
Work with a partner to circle the correct inequality for each situation. The first one is done for you.

<table>
<thead>
<tr>
<th>Real-World Situation</th>
<th>Inequalities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yolanda wants to score at least 84% on the next history test.</td>
<td>$x \leq 84$</td>
</tr>
<tr>
<td>5. To see a certain movie, you must be at least 13 years old.</td>
<td>$n \leq 13$</td>
</tr>
<tr>
<td>Kai has $4.99 left on a music download gift card. She has a download costing $1.99 in her online shopping cart. How much money does Kai have left to spend?</td>
<td>$x + 1.99 \leq 4.99$  $x + 1.99 &gt; 4.99$</td>
</tr>
<tr>
<td>7. In some states, teens must be at least 16 years old to obtain a driver’s license.</td>
<td>$x \leq 16$  $x \geq 16$</td>
</tr>
<tr>
<td>The Walter family budgets a maximum amount of $125 per week for groceries. Mr. Walter already spent $40. How much more can the Walter family spend on groceries?</td>
<td>$x + 40 \leq 125$  $x + 40 &gt; 125$</td>
</tr>
<tr>
<td>Miles pays $30 for a ticket to an amusement park. He cannot spend more than $50. How much more money can Miles spend at the amusement park?</td>
<td>$x + 30 \leq 50$  $x + 30 &gt; 50$</td>
</tr>
</tbody>
</table>

10. **Model with Mathematics** Write a real-world situation that could be represented by $x + 20 \geq 50$.

11. **Inquiry** HOW is an inequality like an equation? How is it different?
Mail  A first class stamp can be used for letters and packages weighing thirteen ounces or less. Fisher is mailing pictures to his grandmother, and only has a first class stamp. His envelope weighs 2 ounces. Follow the steps to determine how much the pictures can weigh so that Fisher can use the stamp.

**Step 1**
Let \( x \) represent the weight of the pictures. Write and solve an equation to find the maximum weight of the pictures.

\[
\text{weight of the envelope} + \ x = \text{maximum weight of the package}
\]

Solve for \( x \).

So, the maximum weight of the pictures is \( \square \) ounces.

**Step 2**
Replace the equals sign in your equation with the less than or equal to symbol, \( \leq \).

\[
2 + x \leq 13
\]

Refer to Step 2. Name three possible values of \( x \) that will result in a true sentence.

---

**Which Mathematical Practices did you use?**

Shade the circle(s) that applies.

- [ ] Persevere with Problems
- [ ] Reason Abstractly
- [ ] Construct an Argument
- [ ] Model with Mathematics
- [ ] Use Math Tools
- [ ] Attend to Precision
- [ ] Make Use of Structure
- [ ] Use Repeated Reasoning
Key Concept

Solve Inequalities

Words
You can solve inequalities by using the **Addition Property of Inequalities** and the **Subtraction Property of Inequalities**. When you add or subtract the same number from each side of an inequality, the inequality remains true.

Symbols
For all numbers \( a, b, \) and \( c, \)
1. If \( a > b, \) then \( a + c > b + c \) and \( a - c > b - c. \)
2. If \( a < b, \) then \( a + c < b + c \) and \( a - c < b - c. \)

Examples
\[
\begin{align*}
2 &< 4 \\
+3 &< +3 \\
5 &< 7 \\
\hline
6 &> 3 \\
-4 &< -4 \\
2 &> -1
\end{align*}
\]

An **inequality** is a mathematical sentence that compares quantities. Solving an inequality means finding values for the variable that make the inequality true.

The table below gives some examples of the words you might use when describing different inequalities.

<table>
<thead>
<tr>
<th>Inequalities</th>
<th>Words</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>* is less than</td>
</tr>
<tr>
<td></td>
<td>* is fewer than</td>
</tr>
<tr>
<td></td>
<td>* is greater than</td>
</tr>
<tr>
<td></td>
<td>* is more than</td>
</tr>
<tr>
<td></td>
<td>* exceeds</td>
</tr>
<tr>
<td></td>
<td>* is less than or equal to</td>
</tr>
<tr>
<td></td>
<td>* is no more than</td>
</tr>
<tr>
<td></td>
<td>* is at most</td>
</tr>
<tr>
<td></td>
<td>* is greater than or equal to</td>
</tr>
<tr>
<td></td>
<td>* is no less than</td>
</tr>
<tr>
<td></td>
<td>* is at least</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Symbols</th>
<th>&lt;</th>
<th>&gt;</th>
<th>( \leq )</th>
<th>( \geq )</th>
</tr>
</thead>
</table>

Examples

1. Solve \( x + 3 > 10. \)

\[
x + 3 > 10 \quad \text{Write the inequality.}
\]

\[
-3 - 3 \quad \text{Subtract 3 from each side.}
\]

\[
x > 7 \quad \text{Simplify.}
\]

Therefore, the solution is \( x > 7. \)

You can check this solution by substituting a number greater than 7 into the original inequality. Try using 8.

Check
\[
x + 3 \quad > 10 \quad \text{Write the inequality.}
\]

\[
8 + 3 \quad \geq \quad 10 \quad \text{Replace} \ x \text{with 8. Is this sentence true?}
\]

\[
11 \quad > \quad 10 \quad \text{This is a true statement.}
\]
2. Solve \(-6 \geq n - 5\).

\[-6 \geq n - 5 \quad \text{Write the inequality.}\n\]
\[+5 \quad +5 \quad \text{Add 5 to each side.}\n\]
\[-1 \geq n \quad \text{Simplify.}\n\]
The solution is \(-1 \geq n\) or \(n \leq -1\).

You can check this solution by substituting \(-1\) or a number less than \(-1\) into the original inequality.

Got it? Do these problems to find out.

Solve each inequality.

a. \(\sigma - 3 < 8\)

b. \(0.4 + y \geq 7\)

Example

3. Solve \(\sigma + \frac{1}{2} < 2\). Graph the solution set on a number line.

\[\sigma + \frac{1}{2} < 2 \quad \text{Write the inequality.}\n\]
\[\frac{1}{2} - \frac{1}{2} \quad \text{Subtract \(\frac{1}{2}\) from each side.}\n\]
\[\sigma < 1 \frac{1}{2} \quad \text{Simplify.}\n\]
The solution is \(\sigma < 1 \frac{1}{2}\).

Check your solution.

Graph the solution.

Got it? Do these problems to find out.

Solve each inequality. Graph the solution set on the number line provided.

\[c. \ h + 4 > 4\]
\[d. \ x - 6 \leq 4\]
**Write Inequalities**

Inequalities can be used to represent real-world situations. You will want to first identify a variable to represent the unknown value.

**Example**

4. Dylan has $18 to ride go-karts and play games at the state fair. Suppose the go-karts cost $5.50. Write and solve an inequality to find the most he can spend on games.

<table>
<thead>
<tr>
<th>Words</th>
<th>Cost of go-kart plus cost of games must be less than or equal to total amount.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Symbols</td>
<td>Let $x$ = the cost of the games.</td>
</tr>
</tbody>
</table>

Inequality \[ 5.5 + x \leq 18 \]

- Write the inequality \((5.50 = 5.5)\)
- Subtract 5.5 from each side.
- \[ x \leq 12.5 \] Simplify.

So, the most Dylan can spend on games is $12.50.

**Guided Practice**

Solve each inequality. Graph the solution set on a number line. (Examples 1–3)

1. \[ 6 + h \geq 12 \]
2. \[ 14 + t > 5 \]

3. An elevator can hold 2,800 pounds or less. Write and solve an inequality that describes how much more weight the elevator can hold if it is currently holding 2,375 pounds. Interpret the solution. (Example 4)

4. Building on the Essential Question Explain when you would use addition and when you would use subtraction to solve an inequality.
**Independent Practice**

**Solve each inequality. (Examples 1 and 2)**

1. \( h - 16 \leq -24 \)
2. \( y + 6 \geq -13 \)
3. \(-3 < n - 8\)
4. \(3 \leq m + 1.4\)
5. \(x + 0.7 > -0.3\)
6. \(w - 8 \geq 5.6\)

**Solve each inequality. Graph the solution set on a number line. (Example 3)**

7. \(m + 5 \geq -1\)
8. \(-11 > t + 7\)

**MP Reason Abstractly Write an inequality and solve each problem. For Exercises 11 and 12, interpret the solution. (Example 4)**

9. Four more than a number is more than 13.
   - Inequality: ___________________________
   - Solution: ___________________________

10. The sum of a number and 19 is at least 8.2.
    - Inequality: ___________________________
    - Solution: ___________________________

11. The high soccer team can have no more than 26 players. Write and solve an inequality to determine how many more players can make the team if the coach has already chosen 17 players.
    - Inequality: ___________________________
    - Solution: ___________________________
    - Interpretation: ___________________________

12. Lalo has 1,500 minutes per month on his cell phone plan. How many more minutes can he use if he has already talked for 785 minutes?
    - Inequality: ___________________________
    - Solution: ___________________________
    - Interpretation: ___________________________
13. Refer to the diagram below.

![Types of Storms Diagram]

a. A hurricane has winds that are at least 74 miles per hour. Suppose a tropical storm has winds that are 42 miles per hour. Write and solve an inequality to find how much the winds must increase before the storm becomes a hurricane.

Inequality: ____________________________ Solution: ____________________________

b. A major storm has wind speeds that are at least 110 miles per hour. Write and solve an inequality that describes how much greater these wind speeds are than the slowest hurricane.

Inequality: ____________________________ Solution: ____________________________

₁₄. **Reason Inductively** Compare and contrast the solutions of

\[ a - 3 = 15 \text{ and } a - 3 \geq 15. \]

₁₅. **Model with Mathematics** Write an addition inequality for the solution set graphed below.

₁₆. **Persevere with Problems** Solve \( x + b > c \) for \( x \).

₁₇. **Reason Inductively** Does the graph shown at the right show the solution set of the inequality \( x + 3 \geq 2 \)? If not, explain how you would change the graph to show the actual solution set.
Solve each inequality.

18. \(10 < b - 8\) \(\Rightarrow 18 < b\)

19. \(1.2 + m \leq 5.5\)

20. \(c - \frac{1}{4} > -2\frac{1}{2}\)

\(\Rightarrow 10 < b - 8\)
\(\Rightarrow + 8 + 8\)
\(\Rightarrow 18 < b\)

MP Model with Mathematics Solve each inequality. Graph the solution set on a number line.

21. \(-21 < a - 16\)

22. \(t - 6.2 < 4\)

Write an inequality and solve each problem.

23. Eight less than a number is less than 10.

Inequality: 
Solution: 

24. The difference between a number and \(21\frac{1}{2}\) is no more than \(14\frac{1}{4}\).

Inequality: 
Solution: 

25. There were a total of 125 cars at a car dealership. A salesperson sold 68 of the cars in one month. Write and solve an inequality that describes how many more cars, at most, the salesman has left to sell. Interpret the solution.

Inequality: 
Solution: 
Interpretation: 

Copy and Solve Solve each inequality. Graph the solution set on a number line. Show your work on a separate sheet of paper.

26. \(n - \frac{1}{5} \leq \frac{3}{10}\)

27. \(6 > x + 3\frac{1}{3}\)

28. \(c + \frac{1}{4} < 5\)

29. \(9 \leq m - 2\frac{1}{5}\)

30. \(\frac{3}{4} + d > 4\frac{1}{2}\)

31. \(-\frac{7}{8} \leq n + 3\frac{5}{16}\)
32. Joaquin can send up to 250 text messages each month. So far this month, he has sent 141 text messages. Let t represent the number of text messages Joaquin can send during the rest of the month.

Write an inequality to model the situation

Solve the inequality for t.

Graph the solution on the number line.

Interpret the solution to the inequality. Explain your reasoning.

33. Which inequality has the solution set shown in the number line below? Select all that apply.

- [ ] $x + 4 \leq 7$
- [ ] $12 > x + 9$
- [ ] $x + 1 \leq 2$
- [ ] $-7 \geq x - 10$

---

**Common Core Spiral Review**

Solve each equation. Then graph each solution on the number line below. 6.EE.6

34. $x + 2 = 1$

35. $x - 1 = -5$

36. $2x = 10$

37. $-2x = 4$

38. $\frac{x}{2} = 1$

39. $\frac{x}{2} = 3$
Real-World Link

Science An astronaut in a space suit weighs about 300 pounds on Earth, but only 50 pounds on the Moon.

<table>
<thead>
<tr>
<th>weight on Earth</th>
<th>weight on Moon</th>
</tr>
</thead>
<tbody>
<tr>
<td>300 lb</td>
<td>50 lb</td>
</tr>
</tbody>
</table>

1. If the astronaut and space suit each weighed half as much, would the inequality still be true?

$$\frac{300}{2} > \frac{50}{2}$$

Divide each side by 2.

Is the inequality still true? Circle yes or no.

Yes No

2. Is the weight of one astronaut greater on Pluto or Earth? Would the weight of 5 astronauts be greater on Pluto or on Earth? Explain by using an inequality.

<table>
<thead>
<tr>
<th>Location</th>
<th>Weight of Astronaut (lb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Earth</td>
<td>300</td>
</tr>
<tr>
<td>Moon</td>
<td>50</td>
</tr>
<tr>
<td>Pluto</td>
<td>67</td>
</tr>
<tr>
<td>Jupiter</td>
<td>796</td>
</tr>
</tbody>
</table>

3. Is the weight of one astronaut greater on Jupiter or on Earth? Would the weight of 5 astronauts be greater on Jupiter or on Earth? Explain by using an inequality.

Which MP Mathematical Practices did you use?
Shade the circle(s) that applies.

1. Persevere with Problems  
2. Reason Abstractly  
3. Construct an Argument  
4. Model with Mathematics  
5. Use Math Tools  
6. Attend to Precision  
7. Make Use of Structure  
8. Use Repeated Reasoning
**Key Concept**

**Multiplication and Division Properties of Inequality, Positive Number**

**Words**

The **Multiplication Property of Inequality** and the **Division Property of Inequality** state that an inequality remains true when you multiply or divide each side of an inequality by a positive number.

**Symbols**

For all numbers $a$, $b$, and $c$, where $c > 0$,

1. If $a > b$, then $ac > bc$ and $\frac{a}{c} > \frac{b}{c}$.
2. If $a < b$, then $ac < bc$ and $\frac{a}{c} < \frac{b}{c}$.

These properties are also true for $a \geq b$ and $a \leq b$.

You can solve inequalities by using the Multiplication Property of Inequality and the Division Property of Inequality.

**Examples**

1. Solve $8x \leq 40$.

   $8x \leq 40$ \hspace{1cm} Write the inequality.

   $\frac{8x}{8} \leq \frac{40}{8}$ \hspace{1cm} Divide each side by 8.

   $x \leq 5$ \hspace{1cm} Simplify.

   The solution is $x \leq 5$. You can check this solution by substituting 5 or a number less than 5 into the inequality.

2. Solve $\frac{d}{2} > 7$.

   $\frac{d}{2} > 7$ \hspace{1cm} Write the inequality.

   $2\left(\frac{d}{2}\right) > 2(7)$ \hspace{1cm} Multiply each side by 2.

   $d > 14$ \hspace{1cm} Simplify.

   The solution is $d > 14$. You can check this solution by substituting a number greater than 14 into the inequality.

**Got it?** Do these problems to find out.

- a. $4x < 40$
- b. $6 \geq \frac{x}{7}$
**Expressions and Equations**

### Multiplication and Division Properties of Inequality, Negative Number

**Words**
When you multiply or divide each side of an inequality by a negative number, the inequality symbol must be reversed for the inequality to remain true.

**Symbols**
For all numbers $a$, $b$, and $c$, where $c < 0$,
1. if $a > b$, then $ac < bc$ and $\frac{a}{c} < \frac{b}{c}$.
2. if $a < b$, then $ac > bc$ and $\frac{a}{c} > \frac{b}{c}$.

**Examples**

- $7 > 1$  \hspace{1cm} $-4 < 16$
- $-2(7) < -2(1)$  \hspace{1cm} Reverse the symbols.  \hspace{1cm} $\frac{-4}{-4} > \frac{16}{-4}$
- $-14 < -2$  \hspace{1cm} $1 > -4$

These properties are also true for $a \geq b$ and $a \leq b$.

### Examples

#### 3. Solve $-2g < 10$. Graph the solution set on a number line.

- $-2g < 10$  \hspace{1cm} Write the inequality.
- $-2g > 10$  \hspace{1cm} Divide each side by $-2$ and reverse the symbol.
- $g > -5$  \hspace{1cm} Simplify.

![Number line diagram]

#### 4. Solve $\frac{x}{3} \leq 4$. Graph the solution set on a number line.

- $\frac{x}{3} \leq 4$  \hspace{1cm} Write the inequality.
- $-3(\frac{x}{3}) \geq -3(4)$  \hspace{1cm} Multiply each side by $-3$ and reverse the symbol.
- $x \geq -12$  \hspace{1cm} Simplify.

![Number line diagram]

### Got it? Do these problems to find out.

- \( c. \ \frac{k}{-2} < 9 \)
Example

5. Ling earns $8 per hour working at the zoo. Write and solve an inequality that can be used to find how many hours she must work in a week to earn at least $120. Interpret the solution.

Words: Amount earned times number is at least amount earned per hour of hours each week.

Variable: Let $x$ represent the number of hours.

Inequality: 

\[8 \cdot x \geq 120\]

- Write the inequality.
- Divide each side by 8.
- Simplify.

So, Ling must work at least 15 hours.

Guided Practice

Solve each inequality. Graph the solution set on a number line. (Examples 1–4)

1. \[-3n \leq -22\]

2. \[\frac{t}{-4} < -11\]

3. At a baseball game you can get a single hot dog for $2. You have $10 to spend. Write and solve an inequality to find the number of hot dogs you can buy. Interpret the solution. (Example 5)

4. Building on the Essential Question: Explain when you should reverse the inequality symbol when solving an inequality.

Rate Yourself!

How confident are you about solving multiplication and division inequalities? Check the box that applies.

For more help, go online to access a Personal Tutor.
Solve each inequality. (Examples 1 and 2)

1. \(6y < 18\)  
2. \(-3s \geq 33\)  
3. \(60 \leq \frac{m}{3}\)  
4. \(\frac{r}{2} < 6\)  
5. \(\frac{m}{-14} \leq -4\)  
6. \(-56 \leq -8x\)

Solve each inequality. Graph the solution set on a number line. (Examples 3 and 4)

10. \(4x \geq 36\)  
11. \(20 < 5t\)  
12. \(\frac{s}{-6} > -16\)  
13. \(\frac{x}{-4} \geq 8\)

14. A pool charges $4 each visit, or you can buy a membership. Write and solve an inequality to find how many times a person should use the pool so that a membership is less expensive than paying each time. Interpret the solution. (Example 5)

Inequality:  
Solution:  
Interpretation:
15. **Reason Inductively** Cross out the inequality that does not belong in the organizer shown at the right. Then explain your reasoning.

\[
\begin{array}{c}
-2x > 12 \\
\frac{x}{2} > -3 \\
-7 < x - 1 \\
-2 < x + 4
\end{array}
\]

Write an inequality for each sentence. Then solve the inequality.

16. Sixteen is less than eight times a number.

   Inequality: 
   Solution: 

17. The product of a number and five is at the most 30.

   Inequality: 
   Solution: 

**H.O.T. Problems** Higher Order Thinking

18. **Identify Structure** Write two different inequalities that have the solution \( y > 6 \). One inequality should be solved using multiplication properties, and the other should be solved using division properties.

19. **Persevere with Problems** You score 15, 16, 17, 14, and 19 points out of 20 possible points on five tests. What must you score on the sixth test to have an average of at least 16 points?

20. **Reason Inductively** The inequalities \( 3x > 2 \) and \( 9x > 6 \) are equivalent inequalities. Write another inequality that is equivalent to \( 3x > 2 \) and \( 9x > 6 \).

21. **Persevere with Problems** Consider the inequalities \( b \geq 4 \) and \( b \leq 13 \).

   a. Graph each inequality on the number line.
   
   b. Do the solution sets of the two inequalities overlap? If so, what does this overlapping area represent?

   c. A compound inequality is an inequality that combines two inequalities. Write a compound inequality for the situation.

   d. Look back at the graph of the solutions for both inequalities. Make another graph that shows only the solution of the compound inequality.
Extra Practice

Solve each inequality.

22. \(-10n > -20\) \(n < 2\)

23. \(-7y < 35\)

24. \(15 < 3r\)

25. \(12p \geq -72\)

26. \(\frac{f}{-7} > 10\)

27. \(-8 < \frac{y}{5}\)

Solve each inequality. Graph the solution set on a number line.

28. \(\frac{h}{5} \leq -12\)

29. \(-3w < -39\)

30. \(15 < 4x\)

31. \(10 \leq \frac{f}{-2}\)

32. **Reason Abstractly** Each game at a carnival costs $0.50, or you can pay $15 and play an unlimited amount of games. Write and solve an inequality to find how many times you should play a game so that the unlimited game play is less expensive than paying each time. Interpret the solution.

   Inequality: ____________________________ Solution: ____________________________

   Interpretation: ____________________________

33. The product of a number and 4 is at least \(-12\).

   Inequality: ____________________________ Solution: ____________________________

34. Five times a number is less than \(-45\).

   Inequality: ____________________________ Solution: ____________________________
35. Caitlin earns $7 per hour babysitting. She wants to earn at least $105 for a camping trip. Determine if each statement is true or false.

- a. The inequality \( \frac{h}{7} \geq 105 \) models how many hours Caitlin must babysit to earn at least $105.  

- True \( \square \)  \( \) False \( \square \)

- b. The inequality \( 7h \geq 105 \) models how many hours Caitlin must babysit to earn at least $105.

- True \( \square \)  \( \) False \( \square \)

- c. Caitlin must babysit up to 15 hours in order to earn at least $105.

- True \( \square \)  \( \) False \( \square \)

36. Soccer balls cost $24 each at Sports Emporium. Coach Neville can spend at most $120 on equipment for the soccer team. Let \( b \) represent the number of soccer balls Coach Neville can buy.

Write an inequality to model the situation. 

Solve the inequality for \( b \). 

Graph the solution on the number line.

How many soccer balls can Coach Neville buy? List all of the possible answers.

37. \( 5k + 6 = 16 \)

38. \(-14 = 2x - 8\)

39. \(-4n + 3 = 13\)

40. \(25 = 7m + 4\)

41. \(10.5 + h = 22.5\)

42. \(14n - 32 = 22\)
Real-World Link

Newspapers: Kaitlyn is placing an ad in the local newspaper for a pottery class. The cost of placing an ad is shown in the table.

<table>
<thead>
<tr>
<th>Service</th>
<th>Cost ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10-day ad with 3 lines</td>
<td>38.00</td>
</tr>
<tr>
<td>each additional line</td>
<td>9.00</td>
</tr>
</tbody>
</table>

1. Complete the equation to find the total cost $c$ of an ad with 4 or more lines. Use $x$ as the variable.

\[
\text{cost of a 10 day ad with only 3 lines} + \text{cost of each additional line} = x
\]

2. How much will it cost to place the ad if it is 5 lines long?

3. Suppose Kaitlyn can spend only $50 on the ad. Does she have enough money to place the ad? Circle yes or no.

   yes  no

   If the answer is no, how much more money will Kaitlyn need? Explain.

Which \textbf{Mathematical Practices} did you use?

Shade the circle(s) that applies.

1. Persevere with Problems
2. Reason Abstractly
3. Construct an Argument
4. Model with Mathematics
5. Use Math Tools
6. Attend to Precision
7. Make Use of Structure
8. Use Repeated Reasoning
Solve a Two-Step Inequality

A **two-step inequality** is an inequality that contains two operations. To solve a two-step inequality, use inverse operations to undo each operation in reverse order of the order of operations.

### Examples

1. Solve $3x + 4 \geq 16$. Graph the solution set on a number line.

   $3x + 4 \geq 16$  
   Write the inequality.

   $-4$  
   Subtract 4 from each side.

   $3x \geq 12$  
   Simplify.

   $\frac{3x}{3} \geq \frac{12}{3}$  
   Divide each side by 3.

   $x \geq 4$  
   Simplify.

   **Graph the solution set.**

   ![Number line with a closed dot at 4 and an arrow to the right]

   Draw a closed dot at 4 with an arrow to the right.

2. Solve $5 + 4x < 33$. Graph the solution set on a number line.

   $5 + 4x < 33$  
   Write the inequality.

   $-5$  
   Subtract 5 from each side.

   $4x < 28$  
   Simplify.

   $\frac{4x}{4} < \frac{28}{4}$  
   Divide each side by 4.

   $x < 7$  
   Simplify.

   **Graph the solution set.**

   ![Number line with an open dot at 7 and an arrow to the left]

   Draw an open dot at 7 with an arrow to the left.

### Got it? Do this problem to find out.

a. Solve $2x + 8 > 24$. Graph the solution on the number line provided.
3. Solve $7 - 2x > 11$. Graph the solution set on a number line.

$7 - 2x > 11$ Write the inequality.
$\frac{-7}{-7}$ Subtract 7 from each side.
$-2x > 4$ Simplify.
$\frac{-2x}{-2} < \frac{4}{-2}$ Divide each side by $-2$. Reverse inequality symbol.
$x < -2$ Simplify. Check your solution.

Graph the solution set. Draw an open dot at $-2$ with an arrow to the left.

You can check the solution by substituting a number less than $-2$ into the original inequality. Try using $-3$.

Check $7 - 2(-3) > 11$ Write the inequality.
$7 - 2(-3) > 11$ Replace $x$ with $-3$. Is the sentence true?
$13 > 11$ This is a true statement. ✓

4. Solve $\frac{x}{2} - 5 < -8$. Graph the solution set on a number line.

$\frac{x}{2} - 5 < -8$ Write the inequality.
$\frac{x}{2} - 5 + 5$ Add 5 to each side.
$\frac{x}{2} < -3$ Simplify.
$\frac{x}{2} \cdot 2 < -3 \cdot 2$ Multiply each side by 2.
$x < -6$ Simplify. Check your solution.

Graph the solution set. Draw an open dot at $-6$ with an arrow to the left.

Got it? Do these problems to find out.

Solve each inequality. Graph the solution set on the number line provided.

b. $\frac{x}{2} + 9 \geq 5$

c. $8 - \frac{x}{3} \leq 7$

Lesson 8 Solve Two-Step Inequalities 515
Example

5. Halfway through the bowling league season, Stewart has 34 strikes. He averages 2 strikes per game. Write and solve an inequality to find how many more games it will take for Stewart to have at least 61 strikes, the league record. Interpret the solution.

The number of strikes plus two strikes per game is at least 61. Let $g$ represent the number of games he needs to bowl.

\[
\begin{align*}
34 + 2g & \geq 61 \\
-34 & \quad -34 \\
2g & \geq 27 \\
\frac{2g}{2} & \geq \frac{27}{2} \\
g & \geq 13.5
\end{align*}
\]

Write the inequality. Subtract 34 from each side. Simplify. Divide each side by 2. Simplify.

Stewart should have at least 61 strikes after 14 more games.

Guided Practice

Solve each inequality. Graph the solution set on a number line. (Examples 1–4)

1. $5x - 7 \geq 43 = \quad 2. 11 \leq 7 + \frac{x}{5}$

3. **Financial Literacy** A rental car company charges $45 plus $0.20 per mile to rent a car. Mr. Lawrence does not want to spend more than $100 for his rental car. Write and solve an inequality to find how many miles he can drive and not spend more than $100. Interpret the solution. (Example 5)

Rate Yourself!

How well do you understand solving two-step inequalities? Circle the image that applies.

For more help, go online to access a Personal Tutor.

516 Chapter 2 Equations and Inequalities
Solve each inequality. Graph the solution set on a number line. (Examples 1-4)

1. \(6x + 14 \geq 20\)  
2. \(4x - 13 < 11\)

3. \(-20 > -2x + 4\)

4. \(\frac{x}{13} + 3 \geq 4\)

5. Tyler needs at least $205 for a new video game system. He has already saved $30. He earns $7 an hour at his job. Write and solve an inequality to find how many hours he will need to work to buy the system. Interpret the solution. (Example 5)

   Inequality: ____________________  Solution: ____________________

   Interpretation: ____________________

6. Three times a number increased by four is less than \(-62\).
7. The quotient of a number and \(-5\) increased by one is at most 7.

8. The quotient of a number and 3 minus two is at least \(-12\).
9. The product of \(-2\) and a number minus six is greater than \(-18\).
Write a two-step inequality that could be represented by each number line.

10. __________________________
11. __________________________

12. __________________________
13. __________________________

### H.O.T. Problems  Higher Order Thinking

14. **Model with Mathematics** Write a real-world example that could be solved by using the inequality $4x + 8 \geq 32$. Then solve the inequality.

15. **Persevere with Problems** In five games, you score 16, 12, 15, 13, and 17 points. Write and solve an inequality to determine how many points you must score in the sixth game to have an average of at least 15 points.

16. **Use Math Tools** Solve $-x + 6 > -(2x + 4)$. Then graph the solution set on the number line.
   
   Solution: __________________________

17. **Model with Mathematics** Write and solve a real-world problem that can be represented by the inequality $4(x - 2.8) \leq 45$. 

---

Chapter 6 Equations and Inequalities
Extra Practice

Solve each inequality. Graph the solution set on a number line.

18. \(4x - 15 \leq 5\)  
   \[\begin{align*}
   4x - 15 & \leq 5 \\
   + 15 & + 15 \\
   4x & \leq 20 \\
   x & \leq 5
   \end{align*}\]

19. \(-73 \geq 15 + 11x\)

20. \(\frac{x}{5} - 2 > 1\)

21. \(9 \leq \frac{x}{14} + 6\)

22. Catie is starting a babysitting business. She spent $26 to make signs to advertise. She charges an initial fee of $5 and then $3 for each hour of service. Write and solve an inequality to find the number of hours she will have to babysit to make a profit. Interpret the solution.
   
   Inequality: 
   Solution: 
   Interpretation:

23. **Reason Abstractly** As a salesperson, Audrey earns $75 per week plus $5 per sale. This week, she wants her pay to be at least $125. Write and solve an inequality for the number of sales Audrey needs to make. Interpret the solution.

   Inequality: 
   Solution: 
   Interpretation:

24. Elijah and his sister went to the movies. They had $34 altogether and spent $9.50 per ticket. Elijah and his sister bought the same snacks. Write and solve an inequality for the amount that each person spent on snacks. Interpret the solution.

   Inequality: 
   Solution: 
   Interpretation:
25. Which of the following are operations that you should use to solve \(-2x - 5 < 7\) for \(x\)? Select all that apply.
- Subtract 7 from both sides.
- Add 5 to both sides.
- Divide both sides by \(-2\).
- Reverse the inequality symbol.

26. The table shows the cost of renting a jet ski.

<table>
<thead>
<tr>
<th>Rental Period</th>
<th>Cost ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>First hour</td>
<td>$55</td>
</tr>
<tr>
<td>Each additional 15-minutes</td>
<td>$10</td>
</tr>
</tbody>
</table>

Jeremy can spend no more than $105 on a jet ski rental. Let \(x\) represent the number of additional 15-minute increments. Fill in each box to write an inequality to represent the situation.

\[
10x + 55 + 10x \leq 105
\]

What is the greatest length of time Jeremy can rent the jet ski?

Common Core Spiral Review

Solve and graph each inequality. 7.EE.4b

27. \(n + 1 > -2\)
Solution: __________

28. \(-2y > 12\)
Solution: __________

29. \(\frac{t}{-1} > -2\)
Solution: __________

Solve each equation. Check your solution. 7.EE.4a

30. \(5y + 6 = 46\)
31. \(-4k - 1 = 47\)
32. \(5 = 8m + 1\)

33. Michael's dad is 30 years of age. He is 2 years more than four times Michael's age \(m\). Write and solve a two-step equation to determine Michael's age. 6.EE.7

Equation: __________  Solution: __________

Need more practice? Download more Extra Practice at connectED.mcgraw-hill.com.
Veterinary Technician

If you love being around animals, enjoy working with your hands, and are good at analyzing problems, a challenging career in veterinary medicine might be a perfect fit for you. Veterinary technicians help veterinarians by helping to diagnose and treat medical conditions. They may work in private clinics, animal hospitals, zoos, aquariums, or wildlife rehabilitation centers.
**Vet Techs Don't Monkey Around**

For each problem, use the information in the tables to write an equation. Then solve the equation.

1. The minimum tail length of an emperor tamarin is 1.6 inches greater than that of a golden lion tamarin. What is the minimum tail length of a golden lion tamarin?

2. The minimum body length of a golden lion tamarin is 5.3 inches less than the maximum body length. What is the maximum body length?

3. Tamarins live an average of 15 years. This is 13 years less than the years that one tamarin in captivity lived. How long did the tamarin in captivity live?

4. The maximum weight of a golden lion tamarin is about 1.97 times the maximum weight of an emperor tamarin. What is the maximum weight of an emperor tamarin? Round to the nearest tenth.

5. For an emperor tamarin, the maximum total length, including the body and tail, is 27 inches. What is the maximum body length of an emperor tamarin?

<table>
<thead>
<tr>
<th>Golden Lion Tamarin Monkeys</th>
<th>Measure</th>
<th>Minimum</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Body length</td>
<td>7.9 in.</td>
<td>t</td>
<td>\ell</td>
</tr>
<tr>
<td>Tail length</td>
<td></td>
<td>13.7 in.</td>
<td>15.7 in.</td>
</tr>
<tr>
<td>Weight</td>
<td>12.7 oz</td>
<td>28 oz</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Emperor Tamarin Monkeys</th>
<th>Measure</th>
<th>Minimum</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Body length</td>
<td>9.2 in.</td>
<td>b</td>
<td></td>
</tr>
<tr>
<td>Tail length</td>
<td>14 in.</td>
<td>16.6 in.</td>
<td></td>
</tr>
<tr>
<td>Weight</td>
<td>10.7 oz</td>
<td>w</td>
<td></td>
</tr>
</tbody>
</table>

**Career Project**

It's time to update your career portfolio! Go to the Occupational Outlook Handbook online and research a career as a veterinary technician. Include brief descriptions of the work environment, education and training requirements, and the job outlook.

Do you think you would enjoy a career as a veterinary technician? Why or why not?

- 
- 
- 
- 

---

522 Chapter 6 Equations
Unscramble each of the clue words.

TOW-SETP
7

PYORERPT
8

DODTIINA
6

NIOSDIV

LABVIERA
5

AILEYQUITN
2

BISTAUTORNC
3

NUATIEQO
1

TIULINTICPOLMA
4

Use the numbered letters to find another vocabulary term from this chapter.

1 2 3 4 5 6 7 8
Got it?

Match each phrase with the correct term.

1. the value of a variable that makes an equation true
   - a. equivalent equations
   - b. equation
   - c. Addition Property of Equality
   - d. coefficient
   - e. formula
   - f. solution

2. the numerical factor in a multiplication expression

3. equations that have the same solution

4. a sentence stating that two quantities are equal

524  Chapter 6  Equations and Inequalities
Fall Reading

Gordon's English teacher assigned a book to be read by October 31st. The students may select a book from the table, and Gordon chose *City Streets*.

<table>
<thead>
<tr>
<th>Book</th>
<th>Number of Pages</th>
</tr>
</thead>
<tbody>
<tr>
<td>City Streets</td>
<td>387</td>
</tr>
<tr>
<td>Life and Time</td>
<td>411</td>
</tr>
<tr>
<td>Myopia</td>
<td>435</td>
</tr>
</tbody>
</table>

Write your answers on another piece of paper. Show all of your work to receive full credit.

**Part A**

By October 19th, Gordon had read 35 pages. Starting on October 20th, he decides to read the same number of pages each day until he finishes the book on October 30th. Write and solve an equation to represent the situation. Let $p$ represent the number of pages read per day. How many pages does Gordon read per day?

**Part B**

Gordon's friend, Kendrick, selected *Myopia*. He read eight pages in class on October 19th and begins reading again on October 23th. He needs to read at least 350 pages by the end of the day on October 28th. Write and solve an inequality to represent this situation and graph the solution on a number line. Let $p$ represent the number of pages read per day. How many pages must Kendrick read per day to accomplish his goal?
Answering the Essential Question

Use what you learned about equations and inequalities to complete the graphic organizer.

When do you use an equals sign?

Essential Question

WHAT does it mean to say two quantities are equal?

When do you use an inequality symbol?

Answer the Essential Question. WHAT does it mean to say two quantities are equal?
UNIT PROJECT

Stand Up and Be Counted

The U.S. Census is used to determine the number of U.S. House of Representative members that each state is assigned. In this project you will:

- **Collaborate** with your classmates as you research Census data and the U.S. House of Representatives.
- **Share** the results of your research in a creative way.
- **Reflect** on how you can communicate mathematical ideas effectively.

**U** Go Online

Work with your group to research and complete each activity. You will use your results in the Share section on the following page.

1. Explore the official U.S. Census website to find the 2010 state populations. There will be interactive maps that display this information. Write down a few facts you find interesting.

2. Create a table that displays the population and the number of U.S. Representatives for your state and three other states. Then create a line plot for the number of U.S. Representatives.

3. Write an equation that uses any state’s population $x$ and its number of U.S. Representative members $y$ to describes the number of people per U.S. Representative $z$.

4. Use your equation from Exercise 3 to determine the approximate number of people per U.S. Representative for the four states you chose. Interpret the results.

5. Look at the 2000 and 2010 census. How did the population of your state and states in your region change? Did the population change affect the number of U.S. Representatives assigned?

6. States can be categorized by population size and density. Write at least two inequalities that compare the states using these categories.
With your group, decide on a way to share what you have learned about the U.S. House of Representatives and state populations. Some suggestions are listed below, but you can also think of other creative ways to present your information. Remember to show how you used mathematics to complete each of the activities in this project!

- Act as a Census representative and create a presentation to encourage people to participate in the census and explain why it is important.
- Write a letter or email to your Representative about what you learned in this project and how it can be used to improve your community.

Check out the note on the right to connect this project with other subjects.

---

7. **Answer the Essential Question** How can you communicate mathematical ideas effectively?

   a. How did you use what you learned about expressions to help you communicate mathematical ideas effectively in this project?

   __________________________

   __________________________

   __________________________

   b. How did you use what you learned about equations and inequalities to help you communicate mathematical ideas effectively in this project?

   __________________________

   __________________________

   __________________________