CHAPTER 4
The sides of $\triangle ABC$ are $\overline{AB}$, $\overline{BC}$, and $\overline{CA}$.
The vertices are points $A$, $B$, and $C$.
The angles are $\angle BAC$ or $\angle A$, $\angle ABC$ or $\angle B$, and $\angle BCA$ or $\angle C$.

Triangles can be classified in two ways – by their *angles* or by their *sides*.

**Example 1:**

a) Classify the triangle as acute, equiangular, obtuse, or right.

b) Classify the triangle as acute, equiangular, obtuse, or right.

c) The frame of this window design is made up of many triangles.

1) Classify $\triangle ACD$.

2) Classify $\triangle ADE$.

**Example 2:** Classify $\triangle XYZ$ as acute, equiangular, obtuse, or right. Explain your reasoning.
To indicate that sides of a triangle are congruent, an equal number of hash marks are drawn on the corresponding sides.

**Example 3:** The triangle truss shown is modeled for steel construction. Classify $\triangle JMN$, $\triangleJKO$, and $\triangle OLN$ as equilateral, isosceles, or scalene.

**Example 4:** If point $Y$ is the midpoint of $VX$, and $WY = 3.0$ units, classify $\triangle VWY$ as equilateral, isosceles, or scalene. Explain your reasoning.

**Example 5:**

a) Find the measures of the sides of isosceles triangle $KLM$ with base $\overline{KL}$.

b) Find the value of $x$ and the measures of each side of an equilateral triangle $ABC$ if $AB = 6x - 8$, $BC = 7 + x$, and $AC = 13 - x$. 
For numbers 1 – 3, classify each triangle as acute, equiangular, obtuse, or right.

1. [Triangle with angles 100°, 40°, 40°]
2. [Triangle with angles 85°, 30°, 65°]
3. [Triangle with angles 30°, 60°, 90°]

For numbers 4 – 7, classify each triangle in the figure at the right by its angles and sides.

4. \( \triangle ABD \)
5. \( \triangle ABC \)
6. \( \triangle EDC \)
7. \( \triangle BDC \)

For numbers 8 and 9, for each triangle, find the value of \( x \) and the measure of each side.

8. \( \triangle FGH \) is an equilateral triangle with \( FG = x + 5 \), \( GH = 3x - 9 \), and \( FH = 2x - 2 \).

9. \( \triangle LMN \) is an isosceles triangle, with \( LM = LN \), \( LM = 3x - 2 \), \( LN = 2x + 1 \), and \( MN = 5x - 2 \).

For numbers 10 – 12, find the measures of the sides of \( \triangle KPL \) and classify each triangle by its sides.

10. \( K(-3, 2) \), \( P(2, 1) \), \( L(-2, -3) \)
11. \( K(5, -3) \), \( P(3, 4) \), \( L(-1, 1) \)

12. \( K(-2, -6) \), \( P(-4, 0) \), \( L(3, -1) \)

13. **DESIGN** Diana entered the design at the right in a logo contest sponsored by a wildlife environmental group. Use a protractor. How many right angles are there?
14. MUSEUMS Paul is standing in front of a museum exhibition. When he turns his head 60° to the left, he can see a statue by Donatello. When he turns his head 60° to the right, he can see a statue by Della Robbia. The two statues and Paul form the vertices of a triangle. Classify this triangle as acute, right, or obtuse.

15. PAPER Marsha cuts a rectangular piece of paper in half along a diagonal. The result is two triangles. Classify these triangles as acute, right, or obtuse.

16. WATERSKIING Kim and Cassandra are waterskiing. They are holding on to ropes that are the same length and tied to the same point on the back of a speed boat. The boat is going full speed ahead and the ropes are fully taut. Kim, Cassandra, and the point where the ropes are tied on the boat form the vertices of a triangle. The distance between Kim and Cassandra is never equal to the length of the ropes. Classify the triangle as equilateral, isosceles, or scalene.

17. BOOKENDS Two bookends are shaped like right triangles. The bottom side of each triangle is exactly half as long as the slanted side of the triangle. If all the books between the bookends are removed and they are pushed together, they will form a single triangle. Classify the triangle that can be formed as equilateral, isosceles, or scalene.

18. DESIGNS Suzanne saw this pattern on a pentagonal floor tile. She noticed many different kinds of triangles were created by the lines on the tile.

a) Identify five triangles that appear to be acute isosceles triangles.

b) Identify five triangles that appear to be obtuse isosceles triangles.
Geometry
Section 4.2 Notes: Angles of Triangles

The Triangle Angle-Sum Theorem can be used to determine the measure of the third angle of a triangle when the other two angle measures are known.

**Theorem 4.1**

**Words** The sum of the measures of the angles of a triangle is 180.

**Example** \(m\angle A + m\angle B + m\angle C = 180\)

**Auxiliary line:** an _____ line or segment drawn in a figure to help ______________ geometry relationships.

**Example 1:** **SOFTBALL** The diagram shows the path of the softball in a drill developed by four players. Find the measure of each numbered angle.

**Exterior Angle:** an _____ formed by one side of the triangle and the extension of an_______ side. Each _________ angle of a triangle has two **remote interior angles** that are _____ adjacent to the exterior angle.

\(\angle 4\) is an exterior angle of \(\triangle ABC\).

Its two remote interior angles are \(\angle 1\) and \(\angle 3\).

**Theorem 4.2**

The measure of an exterior angle of a triangle is equal to the sum of the measures of the two remote interior angles.

**Example** \(m\angle A + m\angle B = m\angle 1\)
A proof uses statements written in boxes and arrows to show the logical progression of an argument. The reason justifying each statement is written below the box. See below for an example.

### Proof  Exterior Angle Theorem

**Given:** \( \triangle ABC \)

**Prove:** \( m\angle A + m\angle B = m\angle 1 \)

**Flow Proof:**

- \( \triangle ABC \)  
  **Given**

- \( \angle 2 \) and \( \angle 1 \) form a linear pair.  
  **Definition of a linear pair**

- \( \angle 2 \) and \( \angle 1 \) are supplementary.  
  **If \( 2 \angle \) form a linear pair, they are supplementary.**

- \( m\angle A + m\angle B + m\angle 2 = 180 \)  
  **Triangle Angle-Sum Theorem**

- \( m\angle 2 + m\angle 1 = 180 \)  
  **Definition of supplementary**

- \( m\angle A + m\angle B + m\angle 2 = m\angle 2 + m\angle 1 \)  
  **Substitution**

- \( m\angle A + m\angle B = m\angle 1 \)  
  **Subtraction Property of Equality**

---

**Example 2: GARDENING** Find the measure of \( \angle FLW \) in the fenced flower garden shown.

---

A corollary is a theorem with a proof that follows as a ____________ of another theorem.

### Corollaries  Triangle Angle-Sum Corollaries

#### 4.1

**Abbreviation:** Acute \( \triangle \) of a rt. \( \triangle \) are comp.

**Example:** If \( \angle C \) is a right angle, then \( \angle A \) and \( \angle B \) are complementary.

#### 4.2

**Example:** If \( \angle L \) is a right or an obtuse angle, then \( \angle J \) and \( \angle K \) must be acute angles.
Example 3: Find the measure of each numbered angle.
Geometry
Section 4.2 Worksheet

For numbers 1 and 2, find the measure of each numbered angle.

1. \[ \text{Angle 1} \]

2. \[ \text{Angle 2} \]

For numbers 3 – 5, find each measure.

3. \( m \angle 1 \)

4. \( m \angle 2 \)

5. \( m \angle 3 \)

For numbers 6 – 9, find each measure.

6. \( m \angle 1 \)

7. \( m \angle 4 \)

8. \( m \angle 3 \)

9. \( m \angle 2 \)

10. \( m \angle 5 \)

11. \( m \angle 6 \)

For numbers 12 and 13, find each measure.

12. \( m \angle 1 \)

13. \( m \angle 2 \)

14. \text{CONSTRUCTION} The diagram shows an example of the Pratt Truss used in bridge construction. Use the diagram to find \( m \angle 1 \).

15. \text{PATHS} Eric walks around a triangular path. At each corner, he records the measure of the angle he creates. He makes one complete circuit around the path. What is the sum of the three angle measures that he wrote down?
16. **STANDING** Sam, Kendra, and Tony are standing in such a way that if lines were drawn to connect the friends they would form a triangle. If Sam is looking at Kendra he needs to turn his head 40° to look at Tony. If Tony is looking at Sam he needs to turn his head 50° to look at Kendra. How many degrees would Kendra have to turn her head to look at Tony if she is looking at Sam?

![Diagram of triangle](image)

17. **TOWERS** A lookout tower sits on a network of struts and posts. Leslie measured two angles on the tower. What is the measure of angle 1?

![Diagram of tower](image)

18. **ZOOS** The zoo lights up the chimpanzee pen with an overhead light at night. The cross section of the light beam makes an isosceles triangle. The top angle of the triangle is 52° and the exterior angle is 116°. What is the measure of angle 1?

![Diagram of chimpanzee](image)

19. **DRAFTING** Chloe bought a drafting table and set it up so that she can draw comfortably from her stool. Chloe measured the two angles created by the legs and the tabletop in case she had to dismantle the table.

   a) Which of the four numbered angles can Chloe determine by knowing the two angles formed with the tabletop? What are their measures?

   ![Diagram of drafting table](image)

   b) What conclusion can Chloe make about the unknown angles before she measures them to find their exact measurements?
If two geometric figures have exactly the same shape and size, they are **congruent**.

<table>
<thead>
<tr>
<th>Congruent</th>
<th>Not Congruent</th>
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<tbody>
<tr>
<td><img src="image1.png" alt="Figure 1" /> <img src="image2.png" alt="Figure 2" /> <img src="image3.png" alt="Figure 3" /></td>
<td><img src="image4.png" alt="Figure 4" /> <img src="image5.png" alt="Figure 5" /> <img src="image6.png" alt="Figure 6" /></td>
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While positioned differently, Figures 1, 2, and 3 are exactly the same shape and size. Figures 4 and 5 are exactly the same shape but not the same size. Figures 5 and 6 are the same size but not exactly the same shape.

In two __________________________, all of the parts of one polygon are congruent to the _____________________ or matching parts of the other polygon. These corresponding parts include **corresponding angles** and **corresponding sides**.

**Key Concept** **Definition of Congruent Polygons**

<table>
<thead>
<tr>
<th>Words</th>
<th>Model</th>
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<tbody>
<tr>
<td>Two polygons are congruent if and only if their corresponding parts are congruent.</td>
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<td>Example</td>
<td>Corresponding Angles</td>
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<td>( \angle A \cong \angle H ) ( \angle B \cong \angle J ) ( \angle C \cong \angle K )</td>
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<td>Corresponding Sides</td>
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<td>( \overline{AB} \cong \overline{HJ} ) ( \overline{BC} \cong \overline{JK} ) ( \overline{AC} \cong \overline{HK} )</td>
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<td>Congruence Statement</td>
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<td>( \triangle ABC \cong \triangle HKJ )</td>
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Other congruence statements for the triangles above exist. Valid congruence statements for congruent polygons list corresponding vertices in the same order.

**Valid Statement**

\[ \triangle BCA \cong \triangle KJH \]

**Not a Valid Statement**

\[ \triangle ABC \cong \triangle HKJ \]

**Example 1:**

a) Show that the polygons are congruent by identifying all of the congruent corresponding parts. Then write a congruence statement.
b) The support beams on the fence form congruent triangles. In the figure \( \triangle ABC \cong \triangle DEF \), which of the following congruence statements correctly identifies corresponding angles or sides?

a) \( \angle ABC \cong \angle EFD \)

b) \( \angle BAC \cong \angle DFE \)

c) \( \overline{BC} \cong \overline{DE} \)

d) \( \overline{AC} \cong \overline{DF} \)

The phrase “if and only if” in the congruent polygon definition means that both the conditional and converse are true. So, if two polygons are congruent, then their corresponding parts are congruent.

For triangles we say *Corresponding Parts of Congruent Triangles are Congruent* or *CPCTC*.

**Example 2:**

a) In the diagram, \( \triangle ITP \cong \triangle NGO \). Find the values of \( x \) and \( y \).

b) In the diagram, \( \triangle FHJ \cong \triangle HFG \). Find the values of \( x \) and \( y \).

**Example 3: ARCHITECTURE** A drawing of a tower’s roof is composed of congruent triangles all converging at a point at the top. If \( \angle IJK \cong \angle IKJ \) and \( m\angle IJK = 72 \), find \( m\angle JIH \).
**Example 4:** Write a two column proof.

**Given:**
\[ \angle L \cong \angle P \]
\[ \overline{LM} \cong \overline{PO} \]
\[ \overline{LN} \cong \overline{PN} \]
\[ \overline{MN} \cong \overline{NO} \]

**Prove:** \( \triangle LMN \cong \triangle PON \)

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**Theorem 4.4 Properties of Triangle Congruence**

- **Property of Triangle Congruence**
  \( \triangle ABC \cong \triangle ABC \)

- **Property of Triangle Congruence**
  If \( \triangle ABC \cong \triangle EFG \), then \( \triangle EFG \cong \triangle ABC \).

- **Property of Triangle Congruence**
  If \( \triangle ABC \cong \triangle EFG \) and \( \triangle EFG \cong \triangle JKL \), then \( \triangle ABC \cong \triangle JKL \).
Geometry

Section 4.3 Worksheet

For numbers 1 and 2, show that the polygons are congruent by identifying all congruent corresponding parts. Then write a congruence statement.

1. 

2. 

For numbers 3 and 4, polygon $ABCD \cong$ polygon $PQRS$.

3. Find the value of $x$.

4. Find the value of $y$.

5. Write a two-column proof.
   Given: $\angle P \cong \angle R$
   $\angle PSQ \cong \angle RSQ$
   $PQ \cong RQ$
   $PS \cong RS$
   Prove: $\triangle PQS \cong \triangle RQS$

6. QUILTING
   a) Indicate the triangles that appear to be congruent.

   b) Name the congruent angles and congruent sides of a pair of congruent triangles.
7. **PICTURE HANGING** Candice hung a picture that was in a triangular frame on her bedroom wall. One day, it fell to the floor, but did not break or bend. The figure shows the object before and after the fall. Label the vertices on the frame after the fall according to the “before” frame.

8. **SIERPINSKI’S TRIANGLE** The figure at the right is a portion of Sierpinski’s Triangle. The triangle has the property that any triangle made from any combination of edges is equilateral. How many triangles in this portion are congruent to the black triangle at the bottom corner?

9. **QUILTING** Stefan drew this pattern for a piece of his quilt. It is made up of congruent isosceles right triangles. He drew one triangle and then repeatedly drew it all the way around. What are the missing measures of the angles of the triangle?

10. **MODELS** Dana bought a model airplane kit. When he opened the box, these two congruent triangular pieces of wood fell out of it. Identify the triangle that is congruent to $\triangle ABC$.

11. **GEOGRAPHY** Igor noticed on a map that the triangle whose vertices are the supermarket, the library, and the post office ($\triangle SLP$) is congruent to the triangle whose vertices are Igor’s home, Jacob’s home, and Ben’s home ($\triangle IJB$). That is, $\triangle SLP \cong \triangle IJB$.

   a) The distance between the supermarket and the post office is 1 mile. Which path along the triangle $\triangle IJB$ is congruent to this?

   b) The measure of $\angle LPS$ is 40°. Identify the angle that is congruent to this angle in $\triangle IJB$. 
In Lesson 4-3, you proved that two triangles were congruent by showing that all six pairs of corresponding parts were congruent. It is possible to prove two triangles congruent using fewer pairs.

**Example 1:** Write a flow proof.

**Given:** \( QU \cong AD \)
\( QD \cong AU \)

**Prove:** \( \triangle QUD \cong \triangle ADU \)

**Example 2:** \( \triangle DVW \) has vertices \( D(-5, -1), V(-1, -2), \) and \( W(-7, -4) \). \( \triangle LPM \) has vertices \( L(1, -5), P(2, -1), \) and \( M(4, -7) \).

**a)** Graph both triangles on the same coordinate plane.

**b)** Use your graph to make a conjecture as to whether the triangles are congruent. Explain your reasoning.
c) Write a logical argument that uses coordinate geometry to support the conjecture you made in part b.

The angle formed by two adjacent sides of a polygon is called an included angle.

### Postulate 4.2

**Words**  
If two sides and the included angle of one triangle are congruent to two sides and the included angle of a second triangle, then the triangles are congruent.

**Example**  
If $\overline{AB} \cong \overline{DE}$,  
Angle $\angle B \cong \angle E$, and  
$\overline{BC} \cong \overline{EF}$,  
then $\triangle ABC \cong \triangle DEF$.

**Example 3:** The wings of one type of moth form two triangles. Write a two-column proof to prove that $\triangle FEG \cong \triangle HIG$ if $\overline{EI} \cong \overline{FH}$, and $G$ is the midpoint of both $\overline{EI}$ and $\overline{FH}$.

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Example 4: Write a paragraph proof.
Given: $RQ \parallel TS$
$\overline{RQ} \cong \overline{TS}$
Prove: $\angle Q \cong \angle S$
Geometry

Section 4.4 Worksheet

Name: _____________________________________

For numbers 1 and 2, determine whether $\triangle DEF \cong \triangle PQR$ given the coordinates of the vertices. Explain.

1. $D(-6, 1), E(1, 2), F(-1, -4), P(0, 5), Q(7, 6), R(5, 0)$

2. $D(-7, -3), E(-4, -1), F(-2, -5), P(2, -2), Q(5, -4), R(0, -5)$

3. Write a flow proof.
   Given: $RS \cong TS$
   $V$ is the midpoint of $RT$
   Prove: $\triangle RSV \cong \triangle TSV$

For numbers 4 – 6, determine which postulate can be used to prove that the triangles are congruent. If it is not possible to prove congruence, write not possible.

4. 

5. 

6. 

7. INDIRECT MEASUREMENT To measure the width of a sinkhole on his property, Harmon marked off congruent triangles as shown in the diagram. How does he know that the lengths $A'B'$ and $AB$ are equal?

8. STICKS Tyson had three sticks of lengths 24 inches, 28 inches, and 30 inches. Is it possible to make two noncongruent triangles using the same three sticks? Explain.
9. **BAKERY** Sonia made a sheet of baklava. She has markings on her pan so that she can cut them into large squares. After she cuts the pastry in squares, she cuts them diagonally to form two congruent triangles. Which postulate could you use to prove the two triangles congruent?

![](image)

10. **CAKE** Carl had a piece of cake in the shape of an isosceles triangle with angles $26^\circ$, $77^\circ$, and $77^\circ$. He wanted to divide it into two equal parts, so he cut it through the middle of the $26^\circ$ angle to the midpoint of the opposite side. He says that because he is dividing it at the midpoint of a side, the two pieces are congruent. Is this enough information? Explain.

11. **TILES** Tammy installs bathroom tiles. Her current job requires tiles that are equilateral triangles and all the tiles have to be congruent to each other. She has a big sack of tiles all in the shape of equilateral triangles. Although she knows that all the tiles are equilateral, she is not sure they are all the same size. What must she measure on each tile to be sure they are congruent? Explain.

12. **INVESTIGATION** An investigator at a crime scene found a triangular piece of torn fabric. The investigator remembered that one of the suspects had a triangular hole in their coat. Perhaps it was a match. Unfortunately, to avoid tampering with evidence, the investigator did not want to touch the fabric and could not fit it to the coat directly.

   a) If the investigator measures all three side lengths of the fabric and the hole, can the investigator make a conclusion about whether or not the hole could have been filled by the fabric?

   b) If the investigator measures two sides of the fabric and the included angle and then measures two sides of the hole and the included angle can he determine if it is a match? Explain.
An **included side** is the side located between two consecutive angles of a polygon. In $\triangle ABC$, $\overline{AC}$ is the included side between $\angle A$ and $\angle C$.

---

**Postulate 4.3**

If two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, then the triangles are congruent.

**Example** If Angle $\angle A \cong \angle D$,

Side $\overline{AB} \cong \overline{DE}$, and

Angle $\angle B \cong \angle E$,

then $\triangle ABC \cong \triangle DEF$.

---

**Example 1**: Write a two-column proof.

Given: $L$ is the midpoint of $\overline{WE}$

$\overline{WR} \parallel \overline{ED}$

Proof: $\triangle WRL \cong \triangle EDL$

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**Theorem 4.5**

If two angles and the nonincluded side of one triangle are congruent to the corresponding two angles and side of a second triangle, then the two triangles are congruent.

**Example** If Angle $\angle A \cong \angle D$,

Angle $\angle B \cong \angle E$, and

Side $\overline{BC} \cong \overline{EF}$,

then $\triangle ABC \cong \triangle DEF$. 
Example 2: Write a paragraph proof.
Given: \( \angle NKL \cong \angle NJM \)
\[ KL \cong JM \]
Proof: \( LN \cong MN \)

You can use congruent triangles to measure distances that are difficult to measure directly.

Example 3: MANUFACTURING Barbara designs a paper template for a certain envelope. She designs the top and bottom flaps to be isosceles triangles that have congruent bases and base angles. If \( EV = 8 \text{ cm} \) and the height of the isosceles triangle is 3 cm, find \( PO \).

Concept Summary

<table>
<thead>
<tr>
<th>SSS</th>
<th>SAS</th>
<th>ASA</th>
<th>AAS</th>
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<tbody>
<tr>
<td>Three pairs of corresponding sides are congruent.</td>
<td>Two pairs of corresponding sides and their included angles are congruent.</td>
<td>Two pairs of corresponding angles and their included sides are congruent.</td>
<td>Two pairs of corresponding angles and the corresponding nonincluded sides are congruent.</td>
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Geometry
Section 4.5 Worksheet

For numbers 1 and 2, write the specified type of proof.

1. Write a flow proof.
   Given: $S$ is the midpoint of $\overline{QT}$
   $\overline{QR} \parallel \overline{TU}$
   Prove: $\triangle QSR \cong \triangle TSU$

2. Write a paragraph proof.
   Given: $\angle D \cong \angle F$
   $\overline{GE}$ bisects $\angle DEF$.
   Prove: $\overline{DG} \cong \overline{FG}$

ARCHITECTURE For numbers 3 and 4, use the following information.
An architect used the window design in the diagram when remodeling an art studio.
$\overline{AB}$ and $\overline{CB}$ each measure 3 feet.

3. Suppose $D$ is the midpoint of $\overline{AC}$. Determine whether $\triangle ABD \cong \triangle CBD$. Justify your answer.

4. Suppose $\angle A \cong \angle C$. Determine whether $\triangle ABD \cong \triangle CBD$. Justify your answer.

5. DOOR STOPS Two door stops have cross-sections that are right triangles. They both have a 20° angle and the length of the side between the 90° and 20° angles are equal. Are the cross sections congruent? Explain.
6. **MAPPING** Two people decide to take a walk. One person is in Bombay and the other is in Milwaukee. They start by walking straight for 1 kilometer. Then they both turn right at an angle of 110°, and continue to walk straight again. After a while, they both turn right again, but this time at an angle of 120°. They each walk straight for a while in this new direction until they end up where they started. Each person walked in a triangular path at their location. Are these two triangles congruent? Explain.

7. **CONSTRUCTION** The rooftop of Angelo’s house creates an equilateral triangle with the attic floor. Angelo wants to divide his attic into 2 equal parts. He thinks he should divide it by placing a wall from the center of the roof to the floor at a 90° angle. If Angelo does this, then each section will share a side and have corresponding 90° angles. What else must be explained to prove that the two triangular sections are congruent?

8. **LOGIC** When Carolyn finished her musical triangle class, her teacher gave each student in the class a certificate in the shape of a golden triangle. Each student received a different shaped triangle. Carolyn lost her triangle on her way home. Later she saw part of a golden triangle under a grate. Is enough of the triangle visible to allow Carolyn to determine that the triangle is indeed hers? Explain.

9. **PARK MAINTENANCE** Park officials need a triangular tarp to cover a field shaped like an equilateral triangle 200 feet on a side.
   a) Suppose you know that a triangular tarp has two 60° angles and one side of length 200 feet. Will this tarp cover the field? Explain.
   
   b) Suppose you know that a triangular tarp has three 60° angles. Will this tarp necessarily cover the field? Explain.
Geometry
Section 4.6 Notes: Isosceles and Equilateral Triangles

The two congruent sides are called the **legs of an isosceles triangle**, and the angle with the sides that are the legs is called the **vertex angle**. The side of the triangle opposite the vertex angle is called the **base**. The two angles formed by the base and the congruent sides are called the **base angles**.

∠1 is the vertex angle.

∠2 and ∠3 are the base angles.

---

**Theorems Isosceles Triangle**

4.10 **Isosceles Triangle Theorem**

Example If \( \overline{AC} \cong \overline{BC} \), then \( \angle 2 \cong \angle 1 \).

4.11 **Converse of Isosceles Triangle Theorem**

If two angles of a triangle are congruent, then the sides opposite those angles are congruent.

---

**Example 1:**

a) Name two unmarked congruent angles.

b) Name two unmarked congruent segments.

c) Which statement correctly names two congruent angles?

a) \( \angle PJM \cong \angle PMJ \)

b) \( \angle JMK \cong \angle JKM \)

c) \( \angle KJP \cong \angle KJP \)

d) \( \angle PML \cong \angle PLK \)
The Isosceles Triangle Theorem leads to two corollaries about the angles of an equilateral triangle.

### Corollaries Equilateral Triangle

**4.3**

**Example** If \( \angle A \cong \angle B \cong \angle C \), then \( AB \cong BC \cong CA \).

![Diagram of an equilateral triangle](image)

**4.4**

**Example** If \( DE \cong EF \cong FE \), then \( m\angle A = m\angle B = m\angle C = 60 \).

![Diagram of another triangle](image)

#### Example 2:

a) Find \( m\angle R \).

b) Find \( PR \).

You can use the properties of equilateral triangles and algebra to find missing values.

#### Example 3: Find the value of each variable.
Example 4: Prove using a two-column proof.
Given: $HEXAGO$ is a regular polygon.
- $\triangle ONG$ is equilateral
- $N$ is the midpoint of $GE$
- $EX \parallel OG$

Prove: $\triangle ENX$ is equilateral.

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
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<td>1.</td>
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<td>13.</td>
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</table>
Geometry
Section 4.6 Worksheet

For numbers 1 – 4, refer to the figure at the right.

1. If \( \overline{RV} \cong \overline{RT} \), name two congruent angles.

2. If \( \overline{RS} \cong \overline{SV} \), name two congruent angles.

3. If \( \angle SRT \cong \angle STR \), name two congruent segments.

4. If \( \angle STV \cong \angle SVT \), name two congruent segments.

For numbers 5 – 9, find each measure.

5. \( m\angle KML \)  

6. \( m\angle HMG \)

7. \( m\angle GHM \)  

8. If \( m\angle HJM = 145^\circ \), find \( m\angle MHJ \)

9. If \( m\angle G = 67^\circ \), find \( m\angle GHM \)

10. Write a two-column proof.

Given: \( DE \parallel BC \)  
\[ \angle 1 \cong \angle 2 \]  
Prove: \( \overline{AB} \cong \overline{AC} \)

11. SPORTS A pennant for the sports teams at Lincoln High School is in the shape of an isosceles triangle. If the measure of the vertex angle is \( 18^\circ \), find the measure of each base angle.
12. **TRIANGLES** At an art supply store, two different triangular rulers are available. One has angles 45°, 45°, and 90°. The other has angles 30°, 60°, and 90°. Which triangle is isosceles?

13. **RULERS** A foldable ruler has two hinges that divide the ruler into thirds. If the ends are folded up until they touch, what kind of triangle results?

14. **HEXAGONS** Juanita placed one end of each of 6 black sticks at a common point and then spaced the other ends evenly around that point. She connected the free ends of the sticks with lines. The result was a regular hexagon. This construction shows that a regular hexagon can be made from six congruent triangles. Classify these triangles. Explain.

15. **PATHS** A marble path is constructed out of several congruent isosceles triangles. The vertex angles are all 20°. What is the measure of angle 1 in the figure?

16. **BRIDGES** Every day, cars drive through isosceles triangles when they go over the Leonard Zakim Bridge in Boston. The ten-lane roadway forms the bases of the triangles.

   a) The angle labeled A in the picture has a measure of 67°. What is the measure of ∠B?

   b) What is the measure of ∠C?

   c) Name the two congruent sides.
A transformation is an operation that maps an original geometric figure, the **preimage**, onto a new figure called the **image**. A transformation can change the position, size, or shape of a figure.

A transformation can be noted using an arrow. The transformation statement \( \triangle ABC \rightarrow \triangle XYZ \) tells you that \( A \) is mapped to \( X \), \( B \) is mapped to \( Y \), and \( C \) is mapped to \( Z \).

A **congruence transformation**, also called a **rigid transformation** or an **isometry**, is one in which the position of the image may differ from that of the preimage, but the two figures remain congruent. The three main types of congruence transformations are shown below.

### KeyConcept

<table>
<thead>
<tr>
<th>Type of Transformation</th>
<th>Description</th>
<th>Example</th>
</tr>
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<tbody>
<tr>
<td><strong>Flip</strong> or <em>flip</em></td>
<td>A transformation over a line called the <em>line of reflection</em>. Each point of the preimage and its image are the same distance from the line of reflection.</td>
<td>( \triangle ABC \rightarrow \triangle FGH )</td>
</tr>
<tr>
<td><strong>Slide</strong> or <em>slide</em></td>
<td>A transformation that moves all points of the original figure the same distance in the same direction.</td>
<td>( \triangle JKL \rightarrow \triangle MPQ )</td>
</tr>
<tr>
<td><strong>Rotation</strong> or <em>turn</em></td>
<td>A transformation around a fixed point called the <em>center of rotation</em>, through a specific angle, and in a specific direction. Each point of the original figure and its image are the same distance from the center.</td>
<td>( \triangle RST \rightarrow \triangle WXY )</td>
</tr>
</tbody>
</table>

**Example 1:** Identify the type of congruence transformation shown as a *reflection*, *translation*, or *rotation*.

a) ![Reflection](image)

b) ![Translation](image)

c) ![Rotation](image)
Example 2:

a) Identify the type of congruence transformation shown by the image of the bridge in the river as a reflection, translation, or rotation.

b) Identify the type of congruence transformation shown by the image of the chess piece as a reflection, translation, or rotation.

You can verify that reflections, translations, and rotations of triangles produce congruent triangles using SSS.

Example 3: Triangle PQR with vertices P(4, 2), Q(3, –3), and R(5, –2) is a transformation of ΔJKL with vertices J(–2, 0), K(–3, –5), and L(–1, –4). Graph the original figure and its image. Identify the transformation and verify that it is a congruence transformation.

Example 4: Triangle ABC with vertices A(–1, –4), B(–4, –1), and C(–1, –1) is a transformation of ΔXYZ with vertices X(–1, 4), Y(–4, 1), and Z(–1, 1). Graph the original figure and its image. Identify the transformation and verify that it is a congruence transformation.
For numbers 1 and 2, identify the type of congruence transformation shown as a reflection, translation, or rotation.

1.  

2.  

3. Identify the type of congruence transformation shown as a reflection, translation, or rotation, and verify that it is a congruence transformation.

4. \( \triangle ABC \) has vertices \( A(-4, 2), B(-2, 0), C(-4, -2) \). \( \triangle DEF \) has vertices \( D(4, 2), E(2, 0), F(4, -2) \). Identify the transformation and verify that it is a congruence transformation.

5. **STENCILS** Carly is planning on stenciling a pattern of flowers along the ceiling in her bedroom. She wants all of the flowers to look exactly the same. What type of congruence transformation should she use? Why?

6. **QUILTING** You and your two friends visit a craft fair and notice a quilt, part of whose pattern is shown below. Your first friend says the pattern is repeated by translation. Your second friend says the pattern is repeated by rotation. Who is correct?
7. **RECYCLING** The international symbol for recycling is shown below. What type of congruence transformation does this illustrate?

![Recycling Symbol]

8. **ANATOMY** Carl notices that when he holds his hands palm up in front of him, they look the same. What type of congruence transformation does this illustrate?

9. **STAMPS** A sheet of postage stamps contains 20 stamps in 4 rows of 5 identical stamps each. What type of congruence transformation does this illustrate?

10. **MOSAICS** José cut rectangles out of tissue paper to create a pattern. He cut out four congruent blue rectangles and then cut out four congruent red rectangles that were slightly smaller to create the pattern shown at the right.

a) Which congruence transformation does this pattern illustrate?

![Mosaic Pattern]

b) If the whole square were rotated 90°, would the transformation still be the same?
Coordinate proofs use figures in the coordinate plane and algebra to prove geometric concepts. The first step in a coordinate proof is placing the figure on the coordinate plane.

Example 1:

a) Position and label right triangle $XYZ$ with leg $XZ$ $d$ units long on the coordinate plane.

b) Which would be the best way to position and label equilateral triangle $ABC$ with side $BC$ $w$ units long on the coordinate plane?

Step 1 Use the origin as a vertex or center of the triangle.
Step 2 Place at least one side of a triangle on an axis.
Step 3 Keep the triangle within the first quadrant if possible.
Step 4 Use coordinates that make computations as simple as possible.
Example 2: Name the missing coordinates of isosceles right triangle $QRS$.

Example 3: Write a coordinate proof to prove that the segment that joins the vertex angle of an isosceles triangle to the midpoint of its base is perpendicular to the base. (Hint: First draw and label an isosceles triangle on the coordinate plane.)

$\textit{Given: } \triangle XYZ \text{ is isosceles.}$

$XW \cong WZ$

$\textit{Prove: } \overline{YW} \perp \overline{XZ}$

Example 4:

a) DRAFTING Write a coordinate proof to prove that the outside of this drafter’s tool is shaped like a right triangle. The length of one side is 10 inches and the length of another side is 5.75 inches.

b) FLAGS Tracy wants to write a coordinate proof to prove this flag is shaped like an isosceles triangle. The altitude is 16 inches and the base is 10 inches.
For numbers 1 – 3, position and label each triangle on the coordinate plane.

1. equilateral $\triangle SWY$ with sides $\frac{1}{4}a$ units long.  

![Diagram of an equilateral triangle with sides labeled $\frac{1}{4}a$ units long.]

2. isosceles $\triangle BLP$ with base $\overline{BL}$ $3b$ units long.  

![Diagram of an isosceles triangle with base labeled $\overline{BL}$ and $3b$ units long.]

3. isosceles right $\triangle DGJ$ with hypotenuse $\overline{DJ}$ and legs $2a$ units long.  

![Diagram of an isosceles right triangle with hypotenuse $\overline{DJ}$ and legs labeled $2a$ units long.]

For numbers 4 – 6, name the missing coordinates of each triangle.

4. $S(?, ?)$  

![Diagram with coordinates labeled $J(0, 0)$, $R(\frac{1}{2}, 0)$, and $S(?, ?)$]

5. $E(0, ?)$  

![Diagram with coordinates labeled $B(-3a, 0)$, $C(?, 0)$, and $E(0, ?)$]

6. $M(0, ?)$  

![Diagram with coordinates labeled $N(?, 0)$, $P(2b, 0)$, and $M(0, ?)$]

**NEIGHBORHOODS** For numbers 7 and 8, use the following information.

Karina lives 6 miles east and 4 miles north of her high school. After school she works part time at the mall in a music store. The mall is 2 miles west and 3 miles north of the school.

7. Write a coordinate proof to prove that Karina’s high school, her home, and the mall are at the vertices of a right triangle.

**Given:** $\triangle SKM$  
**Prove:** $\triangle SKM$ is a right triangle.

![Diagram of a triangle with vertices labeled $S(0, 0)$, $K(8, 4)$, and $M(-2, 3)$]

8. Find the distance between the mall and Karina’s home.
9. **SHELVES** Martha has a shelf bracket shaped like a right isosceles triangle. She wants to know the length of the hypotenuse relative to the sides. She does not have a ruler, but remembers the Distance Formula. She places the bracket on a coordinate grid with the right angle at the origin. The length of each leg is $a$. What are the coordinates of the vertices making the two acute angles?

10. **FLAGS** A flag is shaped like an isosceles triangle. A designer would like to make a drawing of the flag on a coordinate plane. She positions it so that the base of the triangle is on the $y$-axis with one endpoint located at $(0, 0)$. She locates the tip of the flag at \( \left( a, \frac{b}{2} \right) \). What are the coordinates of the third vertex?

11. **BILLIARDS** The figure shows a situation on a billiard table. What are the coordinates of the cue ball before it is hit and the point where the cue ball hits the edge of the table?

12. **TENTS** The entrance to Matt’s tent makes an isosceles triangle. If placed on a coordinate grid with the base on the $x$-axis and the left corner at the origin, the right corner would be at $(6, 0)$ and the vertex angle would be at $(3, 4)$. Prove that it is an isosceles triangle.

13. **DRAFTING** An engineer is designing a roadway. Three roads intersect to form a triangle. The engineer marks two points of the triangle at $(-5, 0)$ and $(5, 0)$ on a coordinate plane.

   a) Describe the set of points in the coordinate plane that could not be used as the third vertex of the triangle.

   b) Describe the set of points in the coordinate plane that would make the vertex of an isosceles triangle together with the two congruent sides.

   c) Describe the set of points in the coordinate plane that would make a right triangle with the other two points if the right angle is located at $(-5, 0)$.
CHAPTER 5
We learned earlier that a segment bisector is any line, segment, or plane that intersects a segment at its ________. If a bisector is also __________________ to the segment, it is called a perpendicular bisector.

**Example 1:**

a) Find the length of $BC$. 

**Theorems Perpendicular Bisectors**

5.1

If a point is on the perpendicular bisector of a segment, then it is equidistant from the endpoints of the segment.

*Example:* If $CD$ is a $\perp$ bisector of $AB$, then $AC = BC$.

5.2

If a point is equidistant from the endpoints of a segment, then it is on the perpendicular bisector of the segment.

*Example:* If $AE = BE$, then $E$ lies on $CD$, the $\perp$ bisector of $AB$. 

Example 1:

a) Find the length of $BC$. 

![Diagram showing a triangle with a perpendicular bisector from C to the midpoint D of AB, with a length of 8.5 units.]
b) Find the length of $XY$.

$$WX = 6$$

$$XY$$

$$YZ$$

$$XZ$$

$$WY$$

$$W$$

$$X$$

$$Y$$

$$Z$$


c) Find the length of $PQ$.

$$PS = 3x + 1$$

$$SQ = 5x - 3$$

When three or more lines intersect at a ________________, the lines are called **concurrent lines**. The point where concurrent lines ___________ is called the **point of concurrency**.

A triangle has three sides, so it also has _______ perpendicular bisectors. These bisectors are concurrent lines. The point of concurrency of the perpendicular bisectors is called the ________________ of the triangle.

**Theorem 5.3 Circumcenter Theorem**

**Words**  
The perpendicular bisectors of a triangle intersect at a point called the ___________ that is equidistant from the vertices of the triangle.

**Example**  
If $P$ is the circumcenter of $\triangle ABC$, then $PB = PA = PC$.

The circumcenter can be on the interior, exterior, or side of a triangle.

- **Acute triangle**
- **Obtuse triangle**
- **Right triangle**
Example 2: A triangular-shaped garden is shown. Can a fountain be placed at the circumcenter and still be inside the garden?

We learned earlier that an angle bisector divides an angle into two congruent angles. The angle bisector can be a line, segment, or ray.

**Theorems  Angle Bisectors**

5.4

If a point is on the bisector of an angle, then it is equidistant from the sides of the angle.

**Example:** If $\overrightarrow{BF}$ bisects $\angle DBE$, $\overrightarrow{FD} \perp \overrightarrow{BD}$, and $\overrightarrow{FE} \perp \overrightarrow{BE}$, then $DF = FE$.

5.5

If a point in the interior of an angle is equidistant from the sides of the angle, then it is on the bisector of the angle.

**Example:** If $\overrightarrow{FD} \perp \overrightarrow{BD}$, $\overrightarrow{FE} \perp \overrightarrow{BE}$, and $DF = FE$, then $\overrightarrow{BF}$ bisects $\angle DBE$.

Example 3:

a) Find the length of $DB$. 
b) Find \( m \angle WYZ \).

![Diagram](image1)

\[ \angle WYZ \]

b) Find \( m \angle WYZ \).

![Diagram](image2)

\[ \angle WYZ \]

The angle bisectors of a triangle are ______________ and their point of concurrency is called the **incenter** of a triangle.

**Theorem 5.6 Incenter Theorem**

**Words** The angle bisectors of a triangle intersect at a point called the __________ that is equidistant from the sides of the triangle.

**Example** If \( P \) is the incenter of \( \triangle ABC \), then \( PD = PE = PF \).

![Diagram](image3)

**Example 4:** Use the diagram to the right.

a) Find \( ST \) if \( S \) is the incenter of \( \triangle MNP \).

![Diagram](image4)

a) Find \( ST \) if \( S \) is the incenter of \( \triangle MNP \).

b) Find \( m \angle SPU \) if \( S \) is the incenter of \( \triangle MNP \).
For numbers 1 – 6, find each measure.

1. $TP$

2. $VU$

3. $KN$

4. $\angle NJZ$

5. $QA$

6. $\angle MFZ$

For numbers 7 & 8, point $L$ is the circumcenter of $\triangle ABC$. List any segment(s) congruent to each segment.

7. $\overline{BN}$

8. $\overline{BL}$

For numbers 9 & 10, point $A$ is the incenter of $\triangle PQR$. Find each measure.

9. $\angle YLA$

10. $\angle YGA$

11. **SCULPTURE** A triangular entranceway has walls with corner angles of $50^\circ$, $70^\circ$, and $60^\circ$. The designer wants to lace a tall bronze sculpture on a round pedestal in a central location equidistant from the three walls. How can the designer find where to place the sculpture?
12. **WIND CHIME** Joanna has a flat wooden triangular piece as part of a wind chime. The piece is suspended by a wire anchored at a point equidistant from the sides of the triangle. Where is the anchor point located?

13. **PICNICS** Marsha and Bill are going to the park for a picnic. The park is triangular. One side of the park is bordered by a river and the other two sides are bordered by busy streets. Marsha and Bill want to find a spot that is equally far away from the river and the streets. At what point in the park should they set up their picnic?

14. **MOVING** Martin has 3 grown children. The figure shows the locations of Martin’s children on a map that has a coordinate plane on it. Martin would like to move to a location that is the same distance from all three of his children. What are the coordinates of the location on the map that is equidistant from all three children?

15. **NEIGHBORHOOD** Amanda is looking at her neighborhood map. She notices that her house along with the homes of her friends, Brian and Cathy, can be the vertices of a triangle. The map is on a coordinate grid. Amanda’s house is at the point (1, 3), Brian’s is at (5, –1), and Cathy’s is at (4, 5). Where would the three friends meet if they each left their houses at the same time and walked to the opposite side of the triangle along the path of shortest distance from their house?

16. **PLAYGROUND** A concrete company is pouring concrete into a triangular form as the center of a new playground.

a) The foreman measures the triangle and notices that the incenter and the circumcenter are the same. What type of triangle is being created?

b) Suppose the foreman changes the triangular form so that the circumcenter is outside of the triangle but the incenter is inside the triangle. What type of triangle would be created?
Geometry
Section 5.2 Notes: Medians and Altitudes of Triangles

A **median** of a triangle is a segment with __________ being a vertex of a triangle and the ________ of the opposite side.

Every triangle has ______ medians that are concurrent. The point of concurrency of the medians of a triangle is called the **centroid** and is ______ inside the triangle.

---

**Theorem 5.7**

The medians of a triangle intersect at a point called the centroid that is two thirds of the distance from each vertex to the midpoint of the opposite side.

**Example** If $P$ is the centroid of $\triangle ABC$, then $AP = \frac{2}{3}AK$, $BP = \frac{2}{3}BL$, and $CP = \frac{2}{3}CJ$.

---

**Example 1:** In $\triangle XYZ$, $P$ is the centroid and $YV = 12$. Find $YP$ and $PV$.

**Example 2:** In $\triangle ABC$, $CG = 4$. Find $GE$.

**Example 3:** **SCULPTURE** An artist is designing a sculpture that balances a triangle on top of a pole. In the artist’s design on the coordinate plane, the vertices are located at $(1, 4)$, $(3, 0)$, and $(3, 8)$. What are the coordinates of the point where the artist should place the pole under the triangle so that it will balance?
An altitude of a triangle is a segment from a _______ to the line containing the opposite side and ______________ to the line containing that side. An altitude can lie in the interior, exterior, or on the side of a triangle.

Example 4: COORDINATE GEOMETRY

The vertices of ΔHIJ are H(1, 2), I(−3, −3), and J(−5, 1). Find the coordinates of the orthocenter of ΔHIJ.
<table>
<thead>
<tr>
<th>Name</th>
<th>Example</th>
<th>Point of Concurrency</th>
<th>Special Property</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>angle bisector</td>
<td><img src="angle_bisector.png" alt="Diagram" /></td>
<td>circumcenter</td>
<td>The circumcenter $P$ of $\triangle ABC$ is equidistant from each vertex.</td>
<td><img src="circumcenter.png" alt="Diagram" /></td>
</tr>
<tr>
<td>median</td>
<td><img src="median.png" alt="Diagram" /></td>
<td>centroid</td>
<td>The incircle $Q$ of $\triangle ABC$ is equidistant from each side of the triangle.</td>
<td><img src="incircle.png" alt="Diagram" /></td>
</tr>
<tr>
<td></td>
<td><img src="orthocenter.png" alt="Diagram" /></td>
<td>orthocenter</td>
<td>The centroid $R$ of $\triangle ABC$ is two thirds of the distance from each vertex to the midpoint of the opposite side.</td>
<td><img src="centroid.png" alt="Diagram" /></td>
</tr>
<tr>
<td></td>
<td><img src="orthocenter.png" alt="Diagram" /></td>
<td>orthocenter</td>
<td>The lines containing the altitudes of $\triangle ABC$ are concurrent at the orthocenter $S$.</td>
<td><img src="orthocenter.png" alt="Diagram" /></td>
</tr>
</tbody>
</table>
For numbers 1 – 6, in $\triangle ABC$, $CP = 30$, $EP = 18$, and $BF = 39$. Find each measure.

1. $PD$
2. $FP$
3. $BP$
4. $CD$
5. $PA$
6. $EA$

For numbers 7 – 12, in $\triangle MIV$, $Z$ is the centroid, $MZ = 6$, $YI = 18$, and $NZ = 12$. Find each measure.

7. $ZR$
8. $YZ$
9. $MR$
10. $ZV$
11. $NV$
12. $IZ$

For numbers 13 & 14, find the coordinates of the centroid of the triangle with the given vertices.

13. $I(3, 1), J(6, 3), K(3, 5)$
14. $H(0, 1), U(4, 3), P(2, 5)$

For numbers 15 & 16, find the coordinates of the orthocenter of the triangle with the given vertices.

15. $P(-1, 2), Q(5, 2), R(2, 1)$
16. $S(0, 0), T(3, 3), U(3, 6)$
17. **MOBILES** Nabuko wants to construct a mobile out of flat triangles so that the surfaces of the triangles hang parallel to the floor when the mobile is suspended. How can Nabuko be certain that she hangs the triangles to achieve this effect?

18. **BALANCING** Johanna balanced a triangle flat on her finger tip. What point of the triangle must Johanna be touching?

19. **REFLECTIONS** Part of the working space in Paulette’s loft is partitioned in the shape of a nearly equilateral triangle with mirrors hanging on all three partitions. From which point could someone see the opposite corner behind his or her reflection in any of the three mirrors?

20. **DISTANCES** For what kind of triangle is there a point where the distance to each side is half the distance to each vertex? Explain.

21. **MEDIANS** Look at the right triangle below. What do you notice about the orthocenter and the vertices of the triangle?

22. **PLAZAS** An architect is designing a triangular plaza. For aesthetic purposes, the architect pays special attention to the location of the centroid $C$ and the circumcenter $O$.

   a) Give an example of a triangular plaza where $C = O$. If no such example exists, state that this is impossible.

   b) Give an example of a triangular plaza where $C$ is inside the plaza and $O$ is outside the plaza. If no such example exists, state that this is impossible.

   c) Give an example of a triangular plaza where $C$ is outside the plaza and $O$ is inside the plaza. If no such example exists, state that this is impossible.
The definition of inequality and the properties of inequalities can be applied to the measures of angles and segments, since these are real numbers. Consider \( \angle 1, \angle 2, \) and \( \angle 3 \) in the figure shown.

By the Exterior Angle Theorem, you know that \( m\angle 1 = m\angle 2 + m\angle 3. \)

Since the angle measures are positive numbers, we can also say that

\[
m\angle 1 > m\angle 2 \quad \text{and} \quad m\angle 1 > m\angle 3
\]

by the definition of inequality.
Example 1: Use the diagram to the right.

a) Use the Exterior Angle Inequality Theorem to list all angles whose measures are less than $m\angle 14$.

b) Use the Exterior Angle Inequality Theorem to list all angles whose measures are greater than $m\angle 5$.

The longest side and largest angle of $\triangle ABC$ are opposite each other. Likewise, the shortest side and smallest angle are opposite each other.

Example 2: List the angles of $\triangle ABC$ in order from smallest to largest.
Example 3: List the sides of $\triangle ABC$ in order from shortest to longest.

Example 4: HAIR ACCESSORIES  Ebony is following directions for folding a handkerchief to make a bandana for her hair. After she folds the handkerchief in half, the directions tell her to tie the two smaller angles of the triangle under her hair. If she folds the handkerchief with the dimensions shown, which two ends should she tie?
Geometry
Section 5.3 Worksheet

Name: _______________________

For numbers 1 – 4, use the figure at the right to determine which angle has the greatest measure.

1. \( \angle 1, \angle 3, \angle 4 \)  
2. \( \angle 4, \angle 8, \angle 9 \)

3. \( \angle 2, \angle 3, \angle 7 \)  
4. \( \angle 7, \angle 8, \angle 10 \)

For numbers 5 – 8, use the Exterior Angle Inequality Theorem to list all angles that satisfy the stated condition.

5. measures are less than \( m\angle 1 \).

6. measures are less than \( m\angle 3 \).

7. measures are greater than \( m\angle 7 \).

8. measures are greater than \( m\angle 2 \).

For numbers 9 – 12, use the figure at the right to determine the relationship between the measures of the given angles.

9. \( m\angle QRW, m\angle RWQ \)  
10. \( m\angle RTW, m\angle TWR \)

11. \( m\angle RST, m\angle TRS \)  
12. \( m\angle WQR, m\angle QRW \)

Use the figure at the right to determine the relationship between the lengths of the given sides.

13. \( \overline{DH}, \overline{GH} \)  
14. \( \overline{DE}, \overline{DG} \)

15. \( \overline{EG}, \overline{FG} \)  
16. \( \overline{DE}, \overline{EG} \)

17. **SPORTS** The figure shows the position of three trees on one part of a disc golf course. At which tree position is the angle between the trees the greatest.

![Diagram of trees and distances]
18. **DISTANCE** Carl and Rose live on the same straight road. From their balconies they can see a flagpole in the distance. The angle that each person’s line of sight to the flagpole makes with the road is the same. How do their distances from the flagpole compare?

19. **OBTUSE TRIANGLES** Don notices that the side opposite the right angle in a right triangle is always the longest of the three sides. Is this also true of the side opposite the obtuse angle in an obtuse triangle? Explain.

20. **STRING** Jake built a triangular structure with three black sticks. He tied one end of a string to vertex $M$ and the other end to a point on the stick opposite $M$, pulling the string taut. Prove that the length of the string cannot exceed the longer of the two sides of the structure.

21. **SQUARES** Matthew has three different squares. He arranges the squares to form a triangle as shown. Based on the information, list the squares in order from the one with the smallest perimeter to the one with the largest perimeter.

22. **CITIES** Stella is going to Texas to visit a friend. As she was looking a map to see where she might want to go, she noticed the cities Austin, Dallas, and Abilene formed a triangle. She wanted to determine how the distances between the cities were related, so she used a protractor to measure two angles.

   a) Based on the information in the figure, which of the two cities are nearest to each other?

   b) Based on the information in the figure, which of the two cities are farthest apart from each other?
Example 1:

a) Is it possible to form a triangle with side lengths of 6.5, 6.5, and 14.5? If not, explain why not.

b) Is it possible to form a triangle with side lengths of 6.8, 7.2, 5.1? If not, explain why not.

Example 2: In \(\triangle PQR\), \(PQ = 7.2\) and \(QR = 5.2\). Which measure cannot be \(PR\)?

a) 7  
b) 9  
c) 11  
d) 13

Example 3: TRAVEL The towns of Jefferson, Kingston, and Newbury are shown in the map below. Prove that driving first from Jefferson to Kingston and then Kingston to Newbury is a greater distance than driving from Jefferson to Newbury.
Geometry
Section 5.5 Worksheet

Name: _____________________________________

For numbers 1 – 8, is it possible to form a triangle with the given side lengths? If not explain why not.

1. 9, 12, 18
2. 8, 9, 17
3. 14, 14, 19
4. 23, 26, 50
5. 32, 41, 63
6. 2.7, 3.1, 4.3
7. 0.7, 1.4, 2.1
8. 12.3, 13.9, 25.2

For numbers 9 – 16, find the range for the measure of the third side of a triangle given the measures of two sides.

9. 6 ft and 19 ft
10. 7 km and 29 km
11. 13 in. and 27 in.
12. 18 ft and 23 ft
13. 25 yd and 38 yd
14. 31 cm and 39 cm
15. 42 m and 6 m
16. 54 in. and 7 in.

17. Given: $H$ is the centroid of $\triangle EDF$.
Prove: $EY + FY > DE$

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $H$ is the centroid of $\triangle EDF$.</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. $\overline{EY}$ is a median.</td>
<td>2. ______________</td>
</tr>
<tr>
<td>3. ______________</td>
<td>3. Definition of median</td>
</tr>
<tr>
<td>4. ______________</td>
<td>4. Definition of midpoint</td>
</tr>
<tr>
<td>5. $EY + DY &gt; DE$</td>
<td>5. ______________</td>
</tr>
<tr>
<td>6. $EY + FY &gt; DE$</td>
<td>6. ______________</td>
</tr>
</tbody>
</table>

18. **GARDENING** Ha Poong has 4 lengths of wood from which he plans to make a border for a triangular-shaped herb garden. The lengths of the wood borders are 8 inches, 10 inches, 12 inches, and 18 inches. How many different triangular borders can Ha Poong make?

19. **STICKS** Jamila has 5 sticks of lengths 2, 4, 6, 8, and 10 inches. Using three sticks at a time as the sides of triangles, how many triangles can she make?
For numbers 20 & 21, use the figure at the right.

20. **PATHS** To get to the nearest super market, Tanya must walk over a railroad track. There are two places where she can cross the track (points A and B). Which path is longer? Explain.

21. **PATHS** While out walking one day Tanya finds a third place to cross the railroad tracks. Show that the path through point C is longer than the path through point B.

22. **CITIES** The distance between New York City and Boston is 187 miles and the distance between New York City and Hartford is 97 miles. Hartford, Boston, and New York City form a triangle on a map. What must the distance between Boston and Hartford be greater than?

23. **TRIANGLES** The figure shows an equilateral triangle \( ABC \) and a point \( P \) outside the triangle.

   a) Draw the figure that is the result of turning the original figure 60° counterclockwise about \( A \). Denote by \( P' \), the image of \( P \) under this turn.

   b) Note that \( P'B \) is congruent to \( PC \). It is also true that \( PP' \) is congruent to \( PA \). Why?

   c) Show that \( PA, PB, \) and \( PC \) satisfy the triangle inequalities.
Example 1:

a) Compare the measures $AD$ and $BD$.

b) Compare the measures of $\angle ABD$ and $\angle BDC$.

Example 2: HEALTH Doctors use a straight-leg-raising test to determine the amount of pain felt in a person’s back. The patient lies flat on the examining table, and the doctor raises each leg until the patient experiences pain in the back area. Nitan can tolerate the doctor raising his right leg 35° and his left leg 65° from the table. Which leg can Nitan raise higher above the table?
When the included angle of one triangle is greater than the included angle in a second triangle, the Converse of the Hinge Theorem is used.

**Example 3:** Find the range of possible values for \( a \).

![Diagram](image1)

**Example 4:** Write a two-column proof.

Given: \( JK = HL \)
\( \overline{JH} \parallel \overline{KL} \)
\( m\angle JKH + m\angle HKL < m\angle JHK + m\angle KHL \)
Prove: \( JH < KL \)

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>1.</td>
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<tr>
<td>2.</td>
<td>2.</td>
</tr>
<tr>
<td>3.</td>
<td>3.</td>
</tr>
<tr>
<td>4.</td>
<td>4.</td>
</tr>
<tr>
<td>5.</td>
<td>5.</td>
</tr>
<tr>
<td>6.</td>
<td>6.</td>
</tr>
<tr>
<td>7.</td>
<td>7.</td>
</tr>
<tr>
<td>8.</td>
<td>8.</td>
</tr>
</tbody>
</table>

**Example 5:** Write a two-column proof.

Given: \( ST = PQ \)
\( SR = QR \)
\( ST = \frac{2}{3} SP \)
Prove: \( m\angle SRP > m\angle PRQ \)

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>1.</td>
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<tr>
<td>2.</td>
<td>2.</td>
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<tr>
<td>3.</td>
<td>3.</td>
</tr>
<tr>
<td>4.</td>
<td>4.</td>
</tr>
<tr>
<td>5.</td>
<td>5.</td>
</tr>
<tr>
<td>6.</td>
<td>6.</td>
</tr>
<tr>
<td>7.</td>
<td>7.</td>
</tr>
</tbody>
</table>
For numbers 1 – 4, compare the given measures.

1. $AB$ and $BK$

2. $ST$ and $SR$

3. $m\angle CDF$ and $m\angle EDF$

4. $m\angle R$ and $m\angle T$

5. **PROOF** Write a two-column proof.
   Given: $G$ is the midpoint of $DF$.
   
   $m\angle 1 > m\angle 2$
   
   Prove: $ED > EF$

6. **TOOLS** Rebecca used a spring clamp to hold together a chair leg she repaired with wood glue. When she opened the clamp, she noticed that the angle between the handles of the clamp decreased as the distance between the handles of the clamp decreased. At the same time, the distance between the gripping ends of the clamp increased. When she released the handles, the distance between the gripping end of the clamp decreased and the distance between the handles increased. Is the clamp an example of the Hinge Theorem or its converse?
7. **CLOCKS** The minute hand of a grandfather clock is 3 feet long and the hour hand is 2 feet long. Is the distance between their ends greater at 3:00 or at 8:00?

8. **FERRIS WHEEL** A Ferris wheel has carriages located at the 10 vertices of a regular decagon. Which carriages are farther away from carriage number 1 than carriage number 4?

9. **WALKWAY** Tyree wants to make two slightly different triangles for his walkway. He has three pieces of wood to construct the frame of his triangles. After Tyree makes the first concrete triangle, he adjusts two sides of the triangle so that the angle they create is smaller than the angle in the first triangle. Explain how this changes the triangle.

10. **MOUNTAIN PEAKS** Emily lives the same distance from three mountain peaks: High Point, Topper, and Cloud Nine. For a photography class, Emily must take a photograph from her house that shows two of the mountain peaks. Which two peaks would she have the best chance of capturing in one image?

11. **RUNNERS** A photographer is taking pictures of three track stars, Amy, Noel, and Beth. The photographer stands on a track, which is shaped like a rectangle with semicircles on both ends.

   a) Based on the information in the figure, list the runners in order from nearest to farthest from the photographer.

   b) Explain how to locate the point along the semicircular curve that the runners are on that is farthest away from the photographer.
CHAPTER 6
A diagonal of a polygon is a segment that connects any two nonconsecutive vertices.

The vertices of polygon \(PQRST\) that are not consecutive with vertex \(P\) are vertices \(R\) and \(S\). Therefore, polygon \(PQRST\) has two diagonals from vertex \(P\), \(PR\) and \(PS\). Notice that the diagonals from vertex \(P\) separate the polygon into three triangles.

The sum of the angle measures of a polygon is the sum of the angle measures of the triangles formed by drawing all the possible diagonals from one vertex.

Since the sum of the angle measures of a triangle is 180°, we can make a table and look for a pattern to find the sum of the angle measures for any convex polygon.

<table>
<thead>
<tr>
<th>Polygon</th>
<th>Number of Sides</th>
<th>Number of Triangles</th>
<th>Sum of Interior Angle Measures</th>
</tr>
</thead>
<tbody>
<tr>
<td>Triangle</td>
<td>3</td>
<td>1</td>
<td>(1)180 or 180</td>
</tr>
<tr>
<td>Quadrilateral</td>
<td>4</td>
<td>2</td>
<td>(2)180 or 360</td>
</tr>
<tr>
<td>Pentagon</td>
<td>5</td>
<td>3</td>
<td>(3)180 or 540</td>
</tr>
<tr>
<td>Hexagon</td>
<td>6</td>
<td>4</td>
<td>(4)180 or 720</td>
</tr>
<tr>
<td>(n)-gon</td>
<td>(n)</td>
<td>(n-2)</td>
<td>((n-2)180)</td>
</tr>
</tbody>
</table>

This leads to the following theorem:

**Theorem 6.1** Polygon Interior Angles Sum

\[
m\angle A + m\angle B + m\angle C + m\angle D + m\angle E = (5 - 2) \cdot 180
\]

\[
= 540
\]

You can use the Polygon Interior Angles Sum Theorem to find the sum of the interior angles of a polygon and to find missing measures in polygons.

**Example 1:** a) Find the sum of the measures of the interior angles of a convex nonagon.

b) Find the measure of each interior angle of parallelogram \(RSTU\).
Recall from Lesson 1.6 that in a regular polygon, all of the interior angles are congruent. You can use this fact and the Polygon Interior Angle Sum Theorem to find the interior angle measure of any regular polygon.

**Example 2:** A mall is designed so that five walkways meet at a food court that is in the shape of a regular pentagon. Find the measure of one of the interior angles of the pentagon.

Given the interior angle measure of a regular polygon, you can also use the Polygon Interior Angles Sum Theorem to find a polygon’s number of sides.

**Example 3:**

a) The measure of an interior angle of a regular polygon is 150°. Find the number of sides in the polygon.

b) The measure of an interior angle of a regular polygon is 144°. Find the number of sides in the polygon.

Does a relationship exist between the number of sides a convex polygon and the sum of its exterior angle measures? Examine the polygons below in which an exterior angle has been measured at each vertex.

Did you notice that the sum of the exterior angle measures in each case is 360°? This suggests the following theorem:

**Theorem 6.2 Polygon Exterior Angles Sum**

\[
m\angle 1 + m\angle 2 + m\angle 3 + m\angle 4 + m\angle 5 + m\angle 6 = 360
\]

**Example 4:**

a) Find the value of \(x\) in the diagram.

b) Find the measure of each exterior angle of a regular decagon.
For numbers 1 – 3, find the sum of the measures of the interior angles of each convex polygon.

1. 11-gon
2. 14-gon
3. 17-gon

For numbers 4 – 6, the measure of an interior angle of a regular polygon is given. Find the number of sides in the polygon.

4. 144
5. 156
6. 160

For numbers 7 & 8, find the measure of each interior angle.

7. \[ \angle J = (2x + 15)^\circ, \quad \angle K = (3x - 20)^\circ, \quad \angle M = (x + 15)^\circ \]

8. \[ \angle R = (6x - 4)^\circ, \quad \angle S = (2x + 8)^\circ \]

For numbers 9 – 14, find the measures of an exterior angle and an interior angle given the number of sides of each regular polygon. Round to the nearest tenth, if necessary.

9. 16
10. 24
11. 30
12. 14
13. 22
14. 40

15. Crystals are classified according to seven crystal systems. The basis of the classification is the shapes of the faces of the crystal. Turquoise belongs to the triclinic system. Each of the six faces of turquoise is in the shape of a parallelogram. Find the sum of the measures of the interior angles of one such face.
16. In the Uffizi gallery in Florence, Italy, there is a room built by Buontalenti called the Tribune (La Tribuna in Italian). This room is shaped like a regular octagon. What angle do consecutive walls of the Tribune make with each other?

17. Jasmine is designing boxes she will use to ship her jewelry. She wants to shape the box like a regular polygon. In order for the boxes to pack tightly, she decides to use a regular polygon that has the property that the measure of its interior angles is half the measure of its exterior angles. What regular polygon should she use?

18. A theater floor plan is shown in the figure. The upper five sides are part of a regular dodecagon. Find $m\angle 1$.

19. Archeologists unearthed parts of two adjacent walls of an ancient castle. Before it was unearthed, they knew from ancient texts that the castle was shaped like a regular polygon, but nobody knew how many sides it had. Some said 6, others 8, and some even said 100. From the information in the figure, how many sides did the castle really have?

20. In Ms. Ricketts’ math class, students made a “polygon path” that consists of regular polygons of 3, 4, 5, and 6 sides joined together as shown.

a) Find $m\angle 2$ and $m\angle 5$.

b) Find $m\angle 3$ and $m\angle 4$.

c) What is $m\angle 1$?
Geometry
Section 6.2 Notes: Parallelograms

A **parallelogram** is a quadrilateral with both pairs of ______________ parallel.

To name a parallelogram, use the symbol \( \square \). In \( \square ABCD \), \( BC \parallel AD \) and \( AB \parallel DC \) by definition.

Other properties of parallelograms are given in the theorems below.

### Theorem  Properties of Parallelograms

**6.3** If a quadrilateral is a **□** then its opposite sides are congruent.

**Abbreviation** \( \text{Opp. sides of a } \square \text{ are } \cong \).

**Example** If \( JKLM \) is a parallelogram, then \( JK \cong ML \) and \( JM \cong KL \).

**6.4** If a quadrilateral is a parallelogram, then its **\( \square \)** are congruent.

**Abbreviation** \( \text{Opp. of a } \square \text{ are } \cong \).

**Example** If \( JKLM \) is a parallelogram, then \( \angle J \cong \angle L \) and \( \angle K \cong \angle M \).

**6.5** If a quadrilateral is a parallelogram, then its consecutive angles are **□**

**Abbreviation** \( \text{Cons. } \angle \text{ in a } \square \text{ are supplementary} \).

**Example** If \( JKLM \) is a parallelogram, then \( x + y = 180 \).

**6.6** If a parallelogram has one right angle, then it has \( \square \).

**Abbreviation** \( \text{If a } \square \text{ has 1 rt. } \angle \text{, it has 4 rt. } \angle \text{s.} \)

**Example** In \( \square JKLM \), if \( \angle J \) is a right angle, then \( \angle K, \angle L, \text{ and } \angle M \) are also right angles.

---

**Example 1:** In parallelogram \( ABCD \), suppose \( m\angle B = 32^\circ \), \( CD = 80 \) inches, and \( BC = 15 \) inches.

a) Find \( AD \).

b) Find \( m\angle C \).

c) Find \( m\angle D \).
The diagonals of a parallelogram have special properties as well.

**Theorem** Diagonals of Parallelograms

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Diag. of a ( \square ) bisect each other.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Example</td>
<td>If ( ABCD ) is a parallelogram, then ( \overline{AP} \cong \overline{PC} ) and ( \overline{DP} \cong \overline{PB} ).</td>
</tr>
</tbody>
</table>

6.7 If a quadrilateral is a parallelogram, then each diagonal separates the parallelogram into two \( \triangle \)

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Diag. separates a ( \square ) into 2 ( \triangle )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Example</td>
<td>If ( ABCD ) is a parallelogram, then ( \triangle ABD \cong \triangle CDB ).</td>
</tr>
</tbody>
</table>

Example 2: a) If \( WXYZ \) is a parallelogram, find the value of \( r \).

\( \overline{WX} \cong _______ \) because…..

\( WX = _______ \) because…..

\( 4r = _______ \) because…..

\( r = _____ \) because…..

b) Find the value of \( s \).

c) Find the value of \( t \).

\( \triangle WXY \cong \) \( \) because…..

\( \angle YWX \cong \) \( \) because…..

\( m\angle YWX = \) \( \) because…..

\( 2t = _____ \) because…..

\( t = _____ \) because…..
You can use Theorem 6.7 to determine the coordinates of the intersection of the diagonals of a parallelogram on a coordinate plan given the coordinates of the vertices.

**Example 3:**

a) What are the coordinates of the intersection of the diagonals of parallelogram MNPR, with vertices M(–3, 0), N(–1, 3), P(5, 4) and R(3, 1)?

b) What are the coordinates of the intersection of the diagonals of parallelogram LMNO, with vertices L(0, –3), M(–2, 1), N(1, 5) and O(3, 1)?

**Example 4:** Write a paragraph proof.

Given: Parallelogram $ABCD$, $\overline{AC}$ and $\overline{BD}$ are diagonals, and point $P$ is the intersection of $\overline{AC}$ and $\overline{BD}$.

Prove: $\overline{AC}$ and $\overline{BD}$ bisect each other.
For numbers 1 – 4, find the value of each variable.

1. 

\[
\begin{align*}
X & \quad 3a - 4 \\
b + 1 & \quad 2b \\
W & \quad a + 2 \\
Y & \quad b + 1
\end{align*}
\]

2. 

\[
\begin{align*}
D & \quad (y + 10)^\circ \\
C & \quad (2y - 40)^\circ \\
A & \quad (y + 10)^\circ \\
B & \quad (4x)^\circ
\end{align*}
\]

3. 

\[
\begin{align*}
E & \quad y + 3 \\
P & \quad y - 3 \\
Q & \quad 2z \\
H & \quad x - 3
\end{align*}
\]

4. 

\[
\begin{align*}
M & \quad y - 8 \\
N & \quad x + 9 \\
L & \quad 3y + 1 \\
O & \quad x - 3
\end{align*}
\]

For numbers 5 – 8, use parallelogram \( RSTU \) to find the measure or value.

5. \( m\angle RST \)

6. \( m\angle STU \)

7. \( m\angle TUR \)

8. \( b \)

For numbers 9 & 10, find the coordinates of the intersection of the diagonals of parallelogram \( PRYZ \) with the given vertices.

9. \( P(2, 5), R(3, 3), Y(-2, -3), Z(-3, -1) \)

10. \( P(2, 3), R(1, -2), Y(-5, -7), Z(-4, -2) \)

11. Write a paragraph proof of the following.
Given: parallelogram \( PRST \) and parallelogram \( PQVU \)
Prove: \( \angle V \cong \angle S \)

12. Mr. Rodriguez used the parallelogram at the right to design a herringbone pattern for a paving stone. He will use the paving stone for a sidewalk. If \( m\angle 1 \) is 130\(^\circ\), find \( m\angle 2, m\angle 3, \) and \( m\angle 4 \).
13. A walkway is made by adjoining four parallelograms as shown. Are the end segments $a$ and $e$ parallel to each other? Explain.

14. Four friends live at the four corners of a block shaped like a parallelogram. Gracie lives 3 miles away from Kenny. How far apart do Teresa and Travis live from each other?

15. Four soccer players are located at the corners of a parallelogram. Two of the players in opposite corners are the goalies. In order for goalie A to be able to see the three others, she must be able to see a certain angle $x$ in her field of vision. What angle does the other goalie have to be able to see in order to keep an eye on the other three players?

16. Make a Venn diagram showing the relationship between squares, rectangles, and parallelograms.

17. On vacation, Tony’s family took a helicopter tour of the city. The pilot said the newest building in the city was the building with this top view. He told Tony that the exterior angle by the front entrance is $72^\circ$. Tony wanted to know more about the building, so he drew this diagram and used his geometry skills to learn a few more things. The front entrance is next to vertex $B$.

a) What are the measures of the four angles of the parallelogram?

b) How many pairs of congruent triangles are there in the figure? What are they?
Geometry
Section 6.3 Notes: Tests for Parallelograms

If a quadrilateral has each pair of opposite sides parallel, it is a parallelogram by definition.

This is not the only test, however, that can be used to determine if a quadrilateral is a parallelogram.

### Theorems  Conditions for Parallelograms

<table>
<thead>
<tr>
<th>Theorem</th>
<th>Conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.9</td>
<td>If both pairs of opposite sides are congruent, then the quadrilateral is a parallelogram.</td>
</tr>
<tr>
<td><strong>Example</strong></td>
<td>If $\overline{AB} \cong \overline{DC}$ and $\overline{AD} \cong \overline{BC}$, then $ABCD$ is a parallelogram.</td>
</tr>
<tr>
<td>6.10</td>
<td>If both pairs of opposite angles of a quadrilateral are congruent, then the quadrilateral is a parallelogram.</td>
</tr>
<tr>
<td><strong>Example</strong></td>
<td>If $\angle A \cong \angle C$ and $\angle B \cong \angle D$, then $ABCD$ is a parallelogram.</td>
</tr>
<tr>
<td>6.11</td>
<td>If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.</td>
</tr>
<tr>
<td><strong>Example</strong></td>
<td>If $\overline{AC}$ and $\overline{BD}$ bisect each other, then $ABCD$ is a parallelogram.</td>
</tr>
<tr>
<td>6.12</td>
<td>If one pair of opposite sides of a quadrilateral are both parallel and congruent, then the quadrilateral is a parallelogram.</td>
</tr>
<tr>
<td><strong>Example</strong></td>
<td>If $\overline{AB} \parallel \overline{DC}$ and $\overline{AB} \cong \overline{DC}$, then $ABCD$ is a parallelogram.</td>
</tr>
</tbody>
</table>

**Example 1:**

a) Determine whether the quadrilateral is a parallelogram. Justify your answer.

b) Which theorem would prove the quadrilateral is a parallelogram?
You can use the conditions of parallelograms to prove relationships in real-world situations.

**Example 2:** Scissor lifts, like the platform lift shown, are commonly applied to tools intended to lift heavy items. In the diagram, \( \angle A \cong \angle C \) and \( \angle B \cong \angle D \). Explain why the consecutive angles will always be supplementary, regardless of the height of the platform.

You can also use the conditions of parallelograms along with algebra to find missing values that make a quadrilateral a parallelogram.

**Example 3:** a) Find \( x \) and \( y \) so that the quadrilateral is a parallelogram.

\[
\overline{AB} \cong \text{_________ because.....}
\]

\[
\overline{AB} = \text{_________ because.....}
\]

\[
4x - 1 = \text{_____________ because....}
\]

b) Find \( m \) so that the quadrilateral is a parallelogram.

**Concept Summary**

**Prove that a Quadrilateral Is a Parallelogram**

- Show that both pairs of opposite sides are parallel. (Definition)
- Show that both pairs of opposite sides are congruent. (Theorem 6.9)
- Show that both pairs of opposite angles are congruent. (Theorem 6.10)
- Show that the diagonals bisect each other. (Theorem 6.11)
- Show that a pair of opposite sides is both parallel and congruent. (Theorem 6.12)
We can use the Distance, Slope, and Midpoint Formulas to determine whether a quadrilateral in the coordinate plane is a parallelogram.

**Example 4:** Quadrilateral $QRST$ has vertices $Q(-1, 3)$, $R(3, 1)$, $S(2, -3)$, and $T(-2, -1)$. Determine whether the quadrilateral is a parallelogram. Justify your answer by using the Slope Formula.

**Example 5:** Write a coordinate proof for the following statement:

If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram.
Geometry
Section 6.3 Worksheet

For numbers 1 – 4, determine whether each quadrilateral is a parallelogram. Justify your answer.

1.  

2.  

3.  

4.  

For numbers 5 & 6, graph each quadrilateral with the given vertices. Determine whether the figure is a parallelogram. Justify your answer with the method indicated.

5.  \( P(-5, 1), S(-2, 2), F(-1, -3), T(2, -2); \) Slope Formula

6.  \( R(-2, 5), O(1, 3), M(-3, -4), Y(-6, -2); \) Distance and Slope Formulas

For numbers 7 – 10, solve for \( x \) and \( y \) so that the quadrilateral is a parallelogram.

7.  

8.  

9.  

10.
11. The pattern shown in the figure is to consist of congruent parallelograms. How can the designer be certain that the shapes are parallelograms?

12. Nikia, Madison, Angela, and Shelby are balancing themselves on an “X”-shaped floating object. To balance themselves, they want to make themselves the vertices of a parallelogram. In order to achieve this, do all four of them have to be the same distance from the center of the object? Explain.

13. Two compass needles placed side by side on a table are both 2 inches long and point due north. Do they form the sides of a parallelogram?

14. Four jets are flying in formation. Three of the jets are shown in the graph. If the four jets are located at the vertices of a parallelogram, what are the three possible locations of the missing jet?

15. When a coordinate plane is placed over the Harrisville town map, the four street lamps in the center are located as shown. Do the four lamps form the vertices of a parallelogram? Explain.

16. Aaron is making a wooden picture frame in the shape of a parallelogram. He has two pieces of wood that are 3 feet long and two that are 4 feet long.

   a) If he connects the pieces of wood at their ends to each other, in what order must he connect them to make a parallelogram?

   b) How many different parallelograms could he make with these four lengths of wood?

   c) Explain something Aaron might do to specify precisely the shape of the parallelogram.
Geometry
Section 6.4 Notes: Rectangles

A rectangle is a parallelogram with ______ right angles. By definition, a rectangle has the following properties.

- All four angles are right angles.
- Opposite sides are parallel and congruent.
- Consecutive angles are supplementary.
- Diagonals bisect each other.

In addition, the diagonals of a rectangle are congruent.

Example 1: a) A rectangular garden gate is reinforced with diagonal braces to prevent it from sagging. If $JK = 12$ feet, and $LN = 6.5$ feet, find $KM$.

$LN = JN$ because…..

$JN + LN = ______$ because…..

$LN + LN = ______$ because…..

$2LN = ______$ because…..

$2(6.5) = ______$ because…..

b) Quadrilateral $EFGH$ is a rectangle. If $GH = 6$ feet and $FH = 15$ feet, find $GJ$.

You can use properties of rectangles along with algebra to find missing values.
Example 2: a) Quadrilateral $RSTU$ is a rectangle. If $m \angle RTU = (8x + 4)^\circ$ and $m \angle SUR = (3x - 2)^\circ$, solve for $x$. 

$m \angle TUR = 90^\circ$ because….

$\overline{PT} \cong \overline{PU}$ because …..

$\angle RTU \cong \angle SUT$ because….

$m \angle RTU = m \angle SUT$ because….

$m \angle SUT + m \angle SUR = 90^\circ$ because……

$m \angle RTU + m \angle SUR = 90^\circ$ because…..

$8x + 4 + 3x - 2 = 90^\circ$ because….

b) Quadrilateral $EFGH$ is a rectangle. If $m \angle FGE = (6x - 5)^\circ$ and $m \angle HFE = (4x - 5)^\circ$, solve for $x$.

The converse of Theorem 6.13 is also true.

**Theorem 6.14 Diagonals of a Rectangle**

If the diagonals of a parallelogram are congruent, then the parallelogram is a rectangle.

**Abbreviation**  
If diag. of a $\square$ are $\cong$, then $\square$ is a rectangle.

**Example**  
If $\overline{WY} \cong \overline{XZ}$ in $\square WXYZ$, then $\square WXYZ$ is a rectangle.

Example 3: Some artists stretch their own canvas over wooden frames. This allows them to customize the size of canvas. In order to ensure that the frame is rectangular before stretching the canvas, an artist measures the sides and the diagonals of the frame. If $AB = 12$ inches, $BC = 35$ inches, $CD = 12$ inches, $DA = 35$ inches, $BD = 37$ inches, and $AC = 37$ inches, explain how an artist can be sure that the frame is rectangular.
You can also use the properties of rectangles to prove that a quadrilateral positioned on a coordinate plane is a rectangle given the coordinates of the vertices.

**Example 4:** Quadrilateral $JKLM$ has vertices $J(-2, 3)$, $K(1, 4)$, $L(3, -2)$, and $M(0, -3)$. Determine whether $JKLM$ is a rectangle using the Distance Formula.
For numbers 1 – 6, quadrilateral $RSTU$ is a rectangle.

1. If $UZ = x + 21$ and $ZS = 3x - 15$, find $US$.

2. If $RZ = 3x + 8$ and $ZS = 6x - 28$, find $UZ$.

3. If $RT = 5x + 8$ and $RZ = 4x + 1$, find $ZT$.

4. If $m\angle SUT = (3x + 6)°$ and $m\angle RUS = (5x - 4)°$, find $m\angle SUT$.

5. If $m\angle SRT = (x + 9)°$ and $m\angle UTR = (2x - 44)°$, find $m\angle UTR$.

6. If $m\angle RSU = (x + 41)°$ and $m\angle TUS = (3x + 9)°$, find $m\angle RSU$.

For numbers 7 – 12, quadrilateral $GHJK$ is a rectangle. Find each measure if $m\angle 1 = 37°$.

7. $m\angle 2$  
8. $m\angle 3$

9. $m\angle 4$  
10. $m\angle 5$

11. $m\angle 6$  
12. $m\angle 7$

For numbers 13 – 15, graph each quadrilateral with the given vertices. Determine whether the figure is a rectangle. Justify your answer using the indicated formula.

13. $B(-4, 3), G(-2, 4), H(1, -2), L(-1, -3)$; Slope Formula

14. $N(-4, 5), O(6, 0), P(3, -6), Q(-7, -1)$; Distance Formula
15. C(0, 5), D(4, 7), E(5, 4), F(1, 2); Slope Formula

16. Huntington Park officials approved a rectangular plot of land for a Japanese Zen garden. Is it sufficient to know that opposite sides of the garden plot are congruent and parallel to determine that the garden plot is rectangular? Explain.

17. Jalen makes the rectangular frame shown. In order to make sure that it is a rectangle, Jalen measures the distances $BD$ and $AC$. How should these two distances compare if the frame is a rectangle?

18. A bookshelf consists of two vertical planks with five horizontal shelves. Are each of the four sections for books rectangles? Explain.

19. A landscaper is marking off the corners of a rectangular plot of land. Three of the corners are in place as shown. What are the coordinates of the fourth corner?

21. Veronica made the pattern shown out of 7 rectangles with four equal sides. The side length of each rectangle is written inside the rectangle.

   a) How many rectangles can be formed using the lines in this figure?

   b) If Veronica wanted to extend her pattern by adding another rectangle with 4 equal sides to make a larger rectangle, what are the possible side lengths of rectangles that she can add?
Geometry  
Section 6.5 Notes: Rhombi and Squares

A rhombus is a parallelogram with all four sides congruent. A rhombus has all the properties of a ____________ and the two additional characteristics described in the theorems below.

**Theorems**  
**Diagonals of a Rhombus**

<table>
<thead>
<tr>
<th>Theorem</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>6.15</strong></td>
<td>If (\square ABCD) is a rhombus, then (\overline{AC} \perp \overline{BD}).</td>
</tr>
<tr>
<td><strong>6.16</strong></td>
<td>If a parallelogram is a rhombus, then each diagonal bisects a pair of opposite angles.</td>
</tr>
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</table>

**Example 1:**

a) The diagonals of rhombus \(WXYZ\) intersect at \(V\). If \(m\angle WZX = 39.5^\circ\), find \(m\angle ZYX\).

\(\overline{ZX}\) bisects \(\angle WZY\) because…..

\(m\angle WZY = 2m\angle WZX\) because…..

\(m\angle QZY = 2(39.5)\) or \(79^\circ\) because…..

\(WZ \parallel XY\) and \(ZY\) is a transversal because…..

\(m\angle WZY + m\angle ZYX = 180^\circ\)

\(79^\circ + m\angle ZYX = 180^\circ\)

b) The diagonals of rhombus \(WXYZ\) intersect at \(V\). If \(WX = 8x - 5\) and \(WZ = 6x + 3\), solve for \(x\).
A square is a parallelogram with four ______ sides and four _____ angles.

Recall that a parallelogram with four right angles is a rectangle, and a parallelogram with four congruent sides is a rhombus. Therefore, a parallelogram that is both a rectangle and a rhombus is also a square.

The Venn diagram summarizes the relationships among parallelograms, rhombi, rectangles, and squares.

All of the properties of parallelograms, rectangles, and rhombi apply to squares. For example, the diagonals of a square bisect each other (parallelogram), are congruent (rectangle), and are perpendicular (rhombus).

You can use properties of rhombi and squares to write proofs.
Example 2: Write a paragraph proof.  
Given: \(LMNP\) is a parallelogram, \(\angle 1 \cong \angle 2\), and \(\angle 2 \cong \angle 6\)  
Prove: \(LMNP\) is a rhombus.

Example 3: Hector is measuring the boundary of a new garden. He wants the garden to be square. He has set each of the corner stakes 6 feet apart. What does Hector need to know to make sure that the garden is square?

Example 4: Determine whether parallelogram \(ABCD\) is a rhombus, a rectangle, or a square for the given vertices: \(A(-2, -1), B(-1, 3), C(3, 2),\) and \(D(2, -2)\). List all that apply. Explain.
Geometry
Section 6.5 Worksheet

For numbers 1 – 4, $PRYZ$ is a rhombus. If $RK = 5$, $RY = 13$ and $m\angle YRZ = 67^\circ$, find each measure.

1. $KY$
2. $PK$
3. $m\angle YKZ$
4. $m\angle PZR$

For numbers 5 – 8, $MNPQ$ is a rhombus. If $PQ = 3\sqrt{2}$ and $AP = 3$, find each measure.

5. $AQ$
6. $m\angle APQ$
7. $m\angle MNP$
8. $PM$

For numbers 9 – 11, use the given set of vertices to determine whether $\square BEFG$ is a rhombus, a rectangle, or a square. List all that apply. Explain.

9. $B(-9, 1), E(2, 3), F(12, -2), G(1, -4)$

10. $B(1, 3), E(7, -3), F(1, -9), G(-5, -3)$

11. $B(-4, -5), E(1, -5), F(-2, -1), G(-7, -1)$
12. The figure is an example of a tessellation. Use a ruler or protractor to measure the shapes and then name the quadrilaterals used to form the figure.

13. A tray rack looks like a parallelogram from the side. The levels for the trays are evenly spaced. What two labeled points form a rhombus with base $AA'$?

14. Charles cuts a rhombus along both diagonals. He ends up with four congruent triangles. Classify these triangles as acute, obtuse, or right.

15. The edges of a window are drawn in the coordinate plane. Determine whether the window is a square or a rhombus.

16. Mackenzie cut a square along its diagonals to get four congruent right triangles. She then joined two of them along their long sides. Show that the resulting shape is a square.

17. Tatianna made the design shown. She used 32 congruent rhombi to create the flower-like design at each corner.
   a) What are the angles of the corner rhombi?
   b) What kinds of quadrilaterals are the dotted and checkered figures?
A **trapezoid** is a quadrilateral with exactly ____ pair of parallel sides. The __________ sides are called bases. The nonparallel sides are called legs. The base angles are formed by the ______ and one of the legs.

In trapezoid $ABCD$, $\angle A$ and $\angle B$ are one pair of base angles and $\angle C$ and $\angle D$ are the other pair.

If the legs of a trapezoid are congruent, then it is an **isosceles trapezoid**.

---

**Example 1:** Each side of the basket shown is an isosceles trapezoid. If $m\angle JML = 130^\circ$, $KN = 6.7$ feet, and $LN = 3.6$ feet.

a) find $m\angle MJK$.

$\overline{JK} \parallel \overline{LM}$ because….

$m\angle JML + m\angle MJK = 180^\circ$ because….

$130^\circ + m\angle MJK = 180^\circ$ because….

$m\angle MJK = ____$ because….

b) find $MN$.

$\overline{JL} \cong \overline{KM}$ because….

$JL = KM$ because….

$10.3 = 6.7 + MN$ because….
Example 2: Quadrilateral $ABCD$ has vertices $A(5, 1)$, $B(-3, -1)$, $C(-2, 3)$, and $D(2, 4)$. Show that $ABCD$ is a trapezoid and determine whether it is an isosceles trapezoid.

The midsegment of a trapezoid is the segment that connects the _______ of the legs of the trapezoid.

The theorem below relates the midsegment and the bases of a trapezoid.

**Theorem 6.24 Trapezoid Midsegment Theorem**

The midsegment of a trapezoid is parallel to each base and its measure is one half the sum of the _______ of the bases.

**Example** If $BE$ is the midsegment of trapezoid $ACDF$, then $AF \parallel BE$, $CD \parallel BE$, and $BE = \frac{1}{2}(AF + CD)$.

Example 3: In the figure, $MN$ is the midsegment of trapezoid $FGJK$. What is the value of $x$?

A kite is a quadrilateral with _______ two pairs of consecutive congruent sides. Unlike a parallelogram, the _______ sides of a kite are not congruent or parallel.
Example 4: a) If $WXYZ$ is a kite, find $m\angle XYZ$.

$\angle WXY \cong \angle WZY$ because.....

$m\angle WZY = _______ \text{ because}....$

$m\angle W + m\angle X + m\angle Y + m\angle Z = _______ \text{ because}....$

b) If $MNPQ$ is a kite, find $NP$.

$(NR)^2 + (MR)^2 = (MN)^2 \text{ because}....$

c) If $BCDE$ is a kite, find $m\angle CDE$. 
For numbers 1 – 4, find each measure.

1. \( m \angle T \)

2. \( m \angle Y \)

3. \( m \angle Q \)

4. \( BC \)

For numbers 5 & 6, use trapezoid \( FEDC \), where \( V \) and \( Y \) are midpoints of the legs.

5. If \( FE = 18 \) and \( VY = 28 \), find \( CD \).

6. If \( m \angle F = 140^\circ \) and \( m \angle E = 125^\circ \), find \( m \angle D \).

For numbers 7 & 8, \( RSTU \) is a quadrilateral with vertices \( R(-3, -3) \), \( S(5, 1) \), \( T(10, -2) \), \( U(-4, -9) \).

7. Verify that \( RSTU \) is a trapezoid.

8. Determine whether \( RSTU \) is an isosceles trapezoid. Explain.

9. A set of stairs leading to the entrance of a building is designed in the shape of an isosceles trapezoid with the longer base at the bottom of the stairs and the shorter base at the top. If the bottom of the stairs is 21 feet wide and the top is 14 feet wide, find the width of the stairs halfway to the top.
10. A carpenter needs to replace several trapezoid-shaped desktops in a classroom. The carpenter knows the lengths of both bases of the desktop. What other measurements, if any, does the carpenter need?

11. Artists use different techniques to make things appear to be 3-dimensional when drawing in two dimensions. Kevin drew the walls of a room. In real life, all of the walls are rectangles. In what shape did he draw the side walls to make them appear 3-dimensional?

12. In order to give the feeling of spaciousness, an architect decides to make a plaza in the shape of a kite. Three of the four corners of the plaza are shown on the coordinate plane. If the fourth corner is in the first quadrant, what are its coordinates?

13. A simplified drawing of the reef runway complex at Honolulu International Airport is shown below. How many trapezoids are there in this image?

14. A light outside a room shines through the door and illuminates a trapezoidal region $ABCD$ on the floor. Under what circumstances would trapezoid $ABCD$ be isosceles?

15. A riser is designed to elevate a speaker. The riser consists of 4 trapezoidal sections that can be stacked one on top of the other to produce trapezoids of varying heights. All of the stages have the same height. If all four stages are used, the width of the top of the riser is 10 feet.

a) If only the bottom two stages are used, what is the width of the top of the resulting riser?

b) What would be the width of the riser if the bottom three stages are used?