GEOMETRY
UNIT 4 WORKBOOK

SPRING 2016
CHAPTER 10: Circles
Common Core State Standards
G.CO.1 – Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along line, and distance around circular arc.
G.C.1 – Prove that all circles are similar.

Mathematical Practices
4 Model with mathematics.
1 Make sense of problems and persevere in solving them.

Learning Targets
• Students will be able to identify and use parts of circles.
• Students will solve problems involving the circumference of a circle.

Section 10.1 Notes: Circles and Circumference

A _________ is the locus or set of all points in a plane equidistant from a given point called the _________ of the circle.

Segments that intersect a circle have special names.

Compare and contrast the pairs of special segments of a circle in the diagram to the right.
Example 1:

a) Name the circle and identify a radius.

b) Identify a chord and a diameter of the circle.

For parts c and d, use the circle on the right.

c) Name the circle and identify a radius.

d) Which segment is not a chord?

IMPORTANT

By definition, the distance from the center of a circle to any point on the circle is always the same. Therefore, all radii \( r \) of a circle are congruent. Since a diameter \( d \) is composed of two radii, all diameters of a circle are also congruent.

Example 2:

a) If \( RT = 21 \) cm, what is the length of \( QV \) ?

b) If \( QS = 26 \) cm, what is the length of \( RV \) ?
As with other figures, pairs of circles can be congruent, similar, or share the other special relationships.

**Example 3:**

a) The diameter of \( \odot X \) is 22 units, the diameter of \( \odot Y \) is 16 units, and \( WZ = 5 \) units. Find \( XY \).

![Diagram showing circles with points X, W, Z, and Y]

b) The diameters of \( \odot D \), \( \odot B \), and \( \odot A \) are 4 inches, 9 inches, and 18 inches. Find \( AC \).

![Diagram showing circles with points D, B, A, and E]
The **circumference** of a circle is the distance around the circle. By definition, the ratio $\frac{C}{d}$ is an irrational number called **$\pi$**. Two formulas for circumference can be derived by using this definition.

<table>
<thead>
<tr>
<th>KeyConcept</th>
<th>Circumference</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Words</strong></td>
<td></td>
</tr>
<tr>
<td><strong>Symbols</strong></td>
<td>$C = \pi d$ or $C = 2\pi r$</td>
</tr>
</tbody>
</table>

**Example 4:**

a) **CROP CIRCLES** A series of crop circles was discovered in Alberta, Canada, on September 4, 1999. The largest of the three circles had a radius of 30 feet. Find its circumference.

b) The Unisphere is a giant steel globe that sits in Flushing Meadows-Corona Park in Queens, New York. It has a diameter of 120 feet. Find its circumference.

**Example 5:**

a) Find the diameter and the radius of a circle to the nearest hundredth if the circumference of the circle is 65.4 feet.

b) Find the radius of a circle to the nearest hundredth if its circumference is 16.8 meters.

A polygon is **inscribed** in a circle if all of its vertices lie on the circle. A circle is circumscribed about polygon if it contains all the vertices of the polygon.

- Quadrilateral $LMNP$ is **inscribed** in $\odot K$.
- Circle $K$ is **circumscribed about** quadrilateral $LMNP$.

c) Find the exact circumference of $\odot K$. 

![Diagram of a circle with a tangent and tangent point]
New Vocabulary: Write the correct term next to each definition. Use the words mentioned today to fill in the blanks.

- ___________ ▶ the distance around the circle
- ___________ ▶ a segment with endpoints at the center and on the circle
- ___________ ▶ the locus or set of all points in a plane equidistant from a given point
- ___________ ▶ two coplanar circles that have the same center
- ___________ ▶ a chord which passes though the center of a circle and is made up of collinear radii
- ___________ ▶ an irrational number which by definition is the ratio of the circumference of a circle to the diameter of the circle
- ___________ ▶ descriptor give to a polygon which is drawn inside a circle such that all of its vertices lie on the circle
- ___________ ▶ the name used to describe the given point in the definition of a circle
- ___________ ▶ a segment with endpoints on the circle
- ___________ ▶ descriptor given to a circle which is drawn about a polygon such that the circle contains all of the vertices of the polygon
- ___________ ▶ two circles with congruent radii

In the below blanks, draw a picture to represent the above vocabulary.

<table>
<thead>
<tr>
<th>Center of a Circle</th>
<th>Radius</th>
<th>Diameter</th>
<th>Chord</th>
</tr>
</thead>
</table>

Challenge Problem

The sum of the circumference of circles H, J, and K shown below is $56\pi$. Find KJ.

Learning Target Checklist

☐ I can identify and use parts of circles.

☐ I can solve problems involving the circumference of a circle.
Geometry
Section 10.1 Worksheet
Name: ______________________

For Exercises 1–7, refer to □L.

1. Name the circle.
2. Name a radius.
3. Name a chord.
4. Name a diameter.
5. Name a radius not drawn as part of a diameter.
6. Suppose the radius of the circle is 3.5 yards. Find the diameter.
7. If $RT = 19$ meters, find $LW$.

The diameters of □L and □M are 20 and 13 units, respectively, and $QR = 4$. Find each measure.

8. $LQ$
9. $RM$

Find the diameter and radius of a circle with the given circumference. Round to the nearest hundredth.

10. $C = 21.2$ ft
11. $C = 5.9$ m

Find the exact circumference of each circle using the given inscribed or circumscribed polygon.

12.
13.

14. **SUNDIALS** Herman purchased a sundial to use as the centerpiece for a garden. The diameter of the sundial is 9.5 inches.
   a. Find the radius of the sundial.
   b. Find the circumference of the sundial to the nearest hundredth.

15. **WHEELS** Zack is designing wheels for a concept car. The diameter of the wheel is 18 inches. Zack wants to make spokes in the wheel that run from the center of the wheel to the rim. In other words, each spoke is a radius of the wheel. How long are these spokes?
16. CAKE CUTTING Kathy slices through a circular cake. The cake has a diameter of 14 inches. The slice that Kathy made is straight and has a length of 11 inches.

Did Kathy cut along a radius, a diameter, or a chord of the circle?

17. COINS Three identical circular coins are lined up in a row as shown.

The distance between the centers of the first and third coins is 3.2 centimeters. What is the radius of one of these coins?

18. PLAZAS A rectangular plaza has a surrounding circular fence. The diagonals of the rectangle pass from one point on the fence through the center of the circle to another point on the fence.

Based on the information in the figure, what is the diameter of the fence? Round your answer to the nearest tenth of a foot.

19. EXERCISE HOOPS Taiga wants to make a circular loop that he can twirl around his body for exercise. He will use a tube that is 2.5 meters long.

a. What will be the diameter of Taiga’s exercise hoop? Round your answer to the nearest thousandth of a meter.

b. What will be the radius of Taiga’s exercise hoop? Round your answer to the nearest thousandth of a meter.
10.2 Warm Up

1. Find the circumference of a circle with a radius of 7.

2. The diameters of Circle F and Circle G are 5 and 6 units, respectively. Find the measure of AB and BF.

Common Core State Standards

G.C.2 Identify and describe relationships among inscribed angles, radii, and chords.

G. C. 5 Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.

Mathematical Practices

6 Attend to precision
4 Model with mathematics

Learning Targets:

- Students will be able to identify central angles, major arcs, minor arcs, and semicircles, and find their measures.
- Students will be able to find arc lengths.

Section 10.2 Notes: Measuring Angles and Arcs

A __________ of a circle is an angle with a vertex in the center of the circle. Its sides contain two radii of the circle.

$\angle ABC$ is a central angle of $\odot B$.

**Key Concept: Sum of Central Angles**

<table>
<thead>
<tr>
<th>Words</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m\angle 1 + m\angle 2 + m\angle 3 = 360$</td>
<td></td>
</tr>
</tbody>
</table>

Example 1:

a) Find the value of $x$.

b) Find the value of $x$.  

$\odot F$
An ______ ______ is a portion of a circle defined by two endpoints. ______ ______ separates the circle into two arcs with measures related to the measure of the central angle.

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**Example 2:** Use the figure on the right.

a) \( WC \) is a radius of \( \odot C \). Identify \( \overline{XZY} \) as a major arc, minor arc, or semicircle. Then find its measure.

![Diagram](image1.png)

b) \( WC \) is a radius of \( \odot C \). Identify \( \overline{WX} \) as a major arc, minor arc, or semicircle. Then find its measure.

![Diagram](image2.png)

c) \( WC \) is a radius of \( \odot C \). Identify \( \overline{XW} \) as a major arc, minor arc, or semicircle. Then find its measure.

![Diagram](image3.png)
are arcs in the same or congruent circles that have the same measure.

**Theorem 10.1**

**Words**

In the same circle or in congruent circles, two minor arcs are congruent if and only if their central angles are congruent.

**Example**

If $\angle 1 \cong \angle 2$, then $\widehat{FG} \cong \widehat{HJ}$. If $\widehat{FG} \cong \widehat{HJ}$, then $\angle 1 \cong \angle 2$.

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**Example 3:**

a) BICYCLES Refer to the circle graph. Find $m\widehat{KL}$.

b) BICYCLES Refer to the circle graph. Find $m\widehat{NJL}$.

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are arcs in a circle that have exactly one point in common.

In $\odot M$, $\widehat{HJ}$ and $\widehat{JK}$ are adjacent arcs. As with adjacent angles, you can add the measures of adjacent arcs.

**Postulate 10.1 Arc Addition Postulate**

**Words**

**Example**

$m\widehat{XYZ} = m\widehat{XY} + m\widehat{YZ}$

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**Example 4:**

a) Find $m\widehat{LHI}$ in $\odot M$.

b) Find the $m\widehat{BCD}$ in $\odot F$. 

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New Vocabulary – Label the diagram with the terms listed at the left.

- **central angle**
- **arc**
- **minor arc**
- **major arc**
- **semicircle**
- **congruent arcs**
- **adjacent arcs**

**arc** is the distance between the endpoints along an arc measured in linear units. Since an arc is a portion of a circle, its length is a fraction of the circumference.

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**Example 5:**

a) Find the length of \( \overline{DA} \). Round to the nearest hundredth.

b) Find the length of \( \overline{DA} \). Round to the nearest hundredth.
Summary of 10.2 – Angles and Arcs

Given the $m \angle EJD = 15$, find each measure in $\odot J$ in the order specified. Justify your answer.

$m\overarc{EFG} = \underline{\hspace{3cm}}$  \hspace{1cm} \hspace{1cm} $m\overarc{ED} = \underline{\hspace{3cm}}$

$m\overarc{GH} = \underline{\hspace{3cm}}$  \hspace{1cm} \hspace{1cm} $m\overarc{EDH} = \underline{\hspace{3cm}}$

Summary of 10.2 – Arc Length

Find the length of $\overarc{BC}$ and $\overarc{BDC}$. The radius of $\odot A$ is 5 centimeters. What is the relationship between the two arc lengths?

$m\overarc{BC} = \underline{\hspace{3cm}}$  \hspace{1cm} \hspace{1cm} $m\overarc{BDC} = \underline{\hspace{3cm}}$

Relationship: \underline{\hspace{3cm}}

Challenge Problem
The measures of arc LM, arc MN and arc NL are in the ratio 5:3:4. Find the measure of each arc.

Learning Target Checklist

- I can identify central angles, major arcs, minor arcs, and semicircles, and find their measures.
- I can find arc lengths.
Geometry
Section 10.2 Worksheet

\[ \overline{AC} \text{ and } \overline{DB} \text{ are diameters of } \bigodot Q. \text{ Identify each arc as a major arc, minor arc, or semicircle of the circle. Then find its measure.} \]

1. \( m\overline{AE} \)  
2. \( m\overline{AB} \)

3. \( m\overline{EDC} \)  
4. \( m\overline{ADC} \)

5. \( m\overline{ABC} \)  
6. \( m\overline{BC} \)

\[ \overline{FH} \text{ and } \overline{EG} \text{ are diameters of } \bigodot P. \text{ Find each measure.} \]

7. \( m\overline{EF} \)  
8. \( m\overline{DE} \)

9. \( m\overline{FG} \)  
10. \( m\overline{DHG} \)

11. \( m\overline{DFG} \)  
12. \( m\overline{DGE} \)

Use \( \bigodot Z \) to find each arc length. Round to the nearest hundredth.

13. \( \overline{QP} \), if \( QZ = 10 \) inches  
14. \( \overline{QR} \), if \( PZ = 12 \) feet

15. \( \overline{FR} \), if \( TR = 15 \) meters  
16. \( \overline{PS} \), if \( ZQ = 7 \) centimeters

17. HOMEWORK Refer to the table, which shows the number of hours students at Leland High School say they spend on homework each night.

   a. If you were to construct a circle graph of the data, how many degrees would be allotted to each category?
   
   b. Describe the arcs associated with each category.

<table>
<thead>
<tr>
<th>Homework</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Less than 1 hour</td>
<td>8%</td>
</tr>
<tr>
<td>1–2 hours</td>
<td>29%</td>
</tr>
<tr>
<td>2–3 hours</td>
<td>58%</td>
</tr>
<tr>
<td>3–4 hours</td>
<td>3%</td>
</tr>
<tr>
<td>Over 4 hours</td>
<td>2%</td>
</tr>
</tbody>
</table>

18. CONDIMENTS A number of people in a park were asked to name their favorite condiment for hot dogs. The results are shown in the circle graph.

What was the second most popular hot dog condiment?
19. CLOCKS Shiatsu is a Japanese massage technique. One of the beliefs is that various body functions are most active at various times during the day. To illustrate this, they use a Chinese clock that is based on a circle divided into 12 equal sections by radii.

What is the measure of any one of the 12 equal central angles?

20. PIES Yolanda has divided a circular apple pie into 4 slices by cutting the pie along 4 radii. The central angles of the 4 slices are $3x$, $6x - 10$, $4x + 10$, and $5x$ degrees. What exactly are the numerical measures of the central angles?

21. RIBBONS Cora is wrapping a ribbon around a cylinder-shaped gift box. The box has a diameter of 15 inches and the ribbon is 60 inches long. Cora is able to wrap the ribbon all the way around the box once, and then continue so that the second end of the ribbon passes the first end. What is the central angle formed between the ends of the ribbon? Round your answer to the nearest tenth of a degree.

22. BIKE WHEELS Lucy has to buy a new wheel for her bike. The bike wheel has a diameter of 20 inches.

   a. If Lucy rolls the wheel one complete rotation along the ground, how far will the wheel travel? Round your answer to the nearest hundredth of an inch.

   b. If the bike wheel is rolled along the ground so that it rotates 45°, how far will the wheel travel? Round your answer to the nearest hundredth of an inch.

   c. If the bike wheel is rolled along the ground for 10 inches, through what angle does the wheel rotate? Round your answer to the nearest tenth of a degree.
Construction Section 10.3

**To construct the perpendicular bisector to $\overline{AB}$, put your compass point on $A$. Open the compass setting to more than half of $\overline{AB}$. Draw an arc above and below $\overline{AB}$. Using the same compass setting, put the point on $B$. Draw an arc above and below $\overline{AB}$. The compass arcs should have intersected above and below $\overline{AB}$. Draw line $l$ through these intersecting arcs.**

Follow similar steps to construct the perpendicular bisector to $\overline{BC}$. Draw line $m$ through a new set of intersecting arcs.

Complete this activity 2 times to create two different size circle.
10.3 Warm Up

1. Find the area of a circle with a diameter of 10 cm.

2. Find the area of the shaded sector. Round to the nearest tenth.

Common Core State Standards
G.C.2 Identify and describe relationships among inscribed angles, radii, and chords.
G.MG.3 Apply geometric methods to solve problems.

Mathematical Practices
4 Model with mathematics
3 Construct viable arguments and critique the reasoning of others

Learning Targets
- Students will be able to recognize and use relationships between arcs and chords.
- Students will be able to recognize and use relationships between arcs, chords, and diameters

Section 10.3 Notes: Arcs and Chords

A _________ is a segment with endpoints on a circle. If a chord is not a diameter, then its endpoints divide the circle into a major and minor arc.

Theorem 10.2

Words

Example

\[ FG \cong HJ \text{ if and only if } FG \cong HJ. \]

Example 1:

a) JEWELRY A circular piece of jade is hung from a chain by two wires around the stone. \( JM \cong KL \) and \( mKL = 90^\circ \). Find \( mJM \).

b) \( \odot W \) has congruent chords \( RS \) and \( TV \). If \( mRS = 85^\circ \), find \( mTV \).
Example 2:

a) ALGEBRA In the figure, \( \overrightarrow{OA} \cong \overrightarrow{OB} \) and \( \overrightarrow{WX} \cong \overrightarrow{YZ} \). Find \( WX \).

![Diagram of circle with labeled points](image)

b) ALGEBRA In the figure, \( \overrightarrow{OG} \cong \overrightarrow{OH} \) and \( \overrightarrow{RT} \cong \overrightarrow{LM} \). Find \( LM \).

![Diagram of circle with labeled points](image)

Example 3:

a) In \( \odot G \), \( m\overrightarrow{DE} = 150^\circ \). Find \( m\overrightarrow{DE} \).

![Diagram of circle with labeled points](image)

b) In \( \odot Z \), \( m\overrightarrow{WX} = 60^\circ \). Find \( m\overrightarrow{UX} \).

![Diagram of circle with labeled points](image)
Example 4:

a) CERAMIC TILE In the ceramic stepping stone below, diameter $AB$ is 18 inches long and chord $EF$ is 8 inches long. Find CD.

b) In the circle below, diameter $QS$ is 14 inches long and chord $RT$ is 10 inches long. Find VU.

Example 5:

a) In $\bigcirc P$, $EF = GH = 24$. Find $PQ$.

b) In $\bigcirc R$, $MN = PO = 29$. Find $RS$. 
Review Vocabulary – In the circle provided, draw a chord which is not a diameter using a bolded line and label it, a diameter using a thin line and label it, and a radius using a dashed line and label it. Identify 3 arcs that were created on the circle.

Challenge Problem

The common chord AB between circle P and circle Q is perpendicular to the segment connecting the centers of the circles. If AB = 10, what is the length of PQ? Explain your reasoning.

Use the diagram below to state Theorems 10.3 and 10.4, from the student book, in your own words.

Theorem 10.3

Theorem 10.4

Learning Target Checklist

- I can recognize and use relationships between arcs and chords.
- I can recognize and use relationships between arcs, chords, and diameter.
ALGEBRA Find the value of $x$ in each circle.

1. \[ \text{Circle with } \angle TUV = 79^\circ, \quad \text{and } \overline{13} = 13\]

2. \[ \text{Circle with } \angle PQR = (x + 17)^\circ, \quad \overline{14} = 14\]

3. \[ \text{Circle with } \angle 38 = 38^\circ, \quad \text{and } \overline{11} = 11\]

4. \[ \text{Circle with } \angle PQR = (4x + 2)^\circ, \quad \text{and } \overline{14} = 14\]

5. \[ \text{Circle with } \angle BAC = 114^\circ, \quad \text{and } \overline{11} = 11\]

In \( \bigcirc Y \) the radius is 34, \( \overline{AB} = 60 \), and \( m\overline{AC} = 71 \). Find each measure.

7. \( m\overline{BC} \)

8. \( m\overline{AB} \)

9. \( \overline{AD} \)

10. \( \overline{BD} \)

11. \( \overline{YD} \)

12. \( \overline{DC} \)

13. In \( \bigcirc U \), \( \overline{VW} = 20 \) and \( \overline{YZ} = 5x \). What is \( x \)?

14. In \( \bigcirc Z \), \( \overline{TR} \cong \overline{TV} \), \( \overline{SZ} = x + 4 \), and \( \overline{UZ} = 2x - 1 \). What is \( x \)?

15. HEXAGON A hexagon is constructed as shown in the figure.

How many different chord lengths occur as side lengths of the hexagon?
16. WATERMARKS For security purposes a jewelry company prints a hidden watermark on the logo of all its official documents. The watermark is a chord located 0.7 cm from the center of a circular ring that has a 2.5 cm radius. To the nearest tenth, what is the length of the chord?

17. ARCHAEOLOGY Only one piece of a broken plate is found during an archaeological dig. Use the sketch of the pottery piece below to demonstrate how constructions with chords and perpendicular bisectors can be used to draw the plate’s original size.

18. CENTERS Neil wants to find the center of a large circle. He draws what he thinks is a diameter of the circle and then marks its midpoint and declares that he has found the center. His teacher asks Neil how he knows that the line he drew is the diameter of the circle and not a smaller chord. Neil realizes that he does not know for sure. What can Neil do to determine if it is an actual diameter?

19. QUILTING Miranda is following directions for a quilt pattern “In a 10-inch diameter circle, measure 3 inches from the center of the circle and mark a chord $\overline{AB}$ perpendicular to the radius of the circle. Then cut along the chord.” Miranda is to repeat this for another chord, $\overline{CD}$. Finally, she is to cut along chord $\overline{DB}$ and $\overline{AC}$. The result should be four curved pieces and one quadrilateral.

   a. If Miranda follows the directions, is she guaranteed that the resulting quadrilateral is a rectangle? Explain.

   b. Assume the resulting quadrilateral is a rectangle. One of the curved pieces has an arc measure of 74. What are the measures of the arcs on the other three curved pieces?
10.4 Warm Up

1. Find the value of x.

2. Find the value of x.

Common Core State Standards
G.C.2 Identify and describe relationships among inscribed angles, radii, and chords.
G.C.3 Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.

Mathematical Practices
7 Look for and make use of structure
3 Construct viable arguments and critique the reasoning of others.

Learning Targets
- Students will be able to find measures of inscribed angles.
- Students will be able to find measures of angles of inscribed polygons.

Section 10.4 Notes: Inscribed Angles

An __________________ has a vertex on a circle and sides that contain chords of the circle. In \( \odot C \), \( \angle QRS \) is an inscribed angle.

An __________________ has endpoints on the sides of an inscribed angle and lies in the interior of the inscribed angle. In \( \odot C \), minor arc \( \widehat{QS} \) is intercepted by \( \angle QRS \).

There are three ways that an angle can be inscribed in a circle.

<table>
<thead>
<tr>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Center ( P ) is on a side of the inscribed angle.</td>
<td>Center ( P ) is inside the inscribed angle.</td>
<td>The center ( P ) is in the exterior of the inscribed angle.</td>
</tr>
</tbody>
</table>

For each of these cases, the following theorem holds true.

Theorem 10.6 Inscribed Angle Theorem

Words: If an angle is inscribed in a circle, then the measure of the angle equals one half the measure of its intercepted arc.

Example: [Blank]
Example 1:

a) Find $m\angle X$.

b) Find $m\overline{XY}$

c) Find $m\angle C$.

d) Find $m\overline{BC}$

Theorem 10.7

Example

$\angle B$ and $\angle C$ both intercept $\overline{AD}$. So, $\angle B \cong \angle C$.

Example 2:

a) Find $m\angle R$.

b) Find $m\angle I$.

Example 3:

a) Write a two-column proof.

Given: $\overline{LO} \cong \overline{MN}$

Prove: $\triangle MNP \cong \triangle LOP$

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>1.</td>
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<tr>
<td>2.</td>
<td>2.</td>
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<tr>
<td>3.</td>
<td>3.</td>
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<tr>
<td>4.</td>
<td>4.</td>
</tr>
<tr>
<td>5.</td>
<td>5.</td>
</tr>
<tr>
<td>6.</td>
<td>6.</td>
</tr>
</tbody>
</table>
b) Write a two-column proof.
Given: $\overset{\frown}{AB} \cong \overset{\frown}{CD}$
Prove: $\triangle ABE \cong \triangle DCE$

Select the appropriate reason that goes in the blank to complete the proof below.

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $\overset{\frown}{AB} \cong \overset{\frown}{CD}$</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. $\overset{\frown}{AB} \cong \overset{\frown}{CD}$</td>
<td>2. If minor arcs are congruent, then corresponding chords are congruent.</td>
</tr>
<tr>
<td>3. $\angle D$ intercepts $\overset{\frown}{BC}$ and $\angle A$ intercepts $\overset{\frown}{BC}$.</td>
<td>3. Definition of intercepted arc</td>
</tr>
<tr>
<td>4. $\angle D \cong \angle A$</td>
<td>4. Inscribed angles of the same arc are congruent.</td>
</tr>
<tr>
<td>5. $\angle DEC \cong \angle BEA$</td>
<td>5. Vertical angles are congruent.</td>
</tr>
<tr>
<td>6. $\triangle ABE \cong \triangle DCE$</td>
<td>6. _________________</td>
</tr>
</tbody>
</table>

**Example 4:**

a) Find $m \angle B$.

b) Find $m \angle D$.

**Theorem 10.8**

**Words**

**Example** If $\overset{\frown}{FJH}$ is a semicircle, then $m \angle G = 90$. If $m \angle G = 90$, then $\overset{\frown}{FJH}$ is a semicircle and $FH$ is a diameter.

**Theorem 10.9**

**Words**

**Example** If quadrilateral $KLMN$ is inscribed in $\odot A$, then $\angle L$ and $\angle N$ are supplementary and $\angle K$ and $\angle M$ are supplementary.
Example 5:

a) INSIGNIAS An insignia is an emblem that signifies rank, achievement, membership, and so on. The insignia shown is a quadrilateral inscribed in a circle. Find $m \angle S$ and $m \angle T$.

b) INSIGNIAS An insignia is an emblem that signifies rank, achievement, membership, and so on. The insignia shown is a quadrilateral inscribed in a circle. Find $m \angle N$.

New Vocabulary Write the definition next to each term.

- inscribed angle
- intercepted arc

Writing in math -
A 45-45-90 right triangle is inscribed in a circle. If the radius of the circle is given, explain how to find the lengths of the right triangle’s legs.

Challenge Problem
A square is inscribed in a circle. What is the ratio of the area of the circle to the area of the square?

Learning Target Checklist

- [ ] I can find measures of inscribed angles.
- [ ] I can find measures of angles of inscribed polygons.
Geometry
Section 10.4 Worksheet

Find each measure.

1. \( m\overline{XY} \)

2. \( m\angle E \)

3. \( m\angle R \)

4. \( m\overline{MP} \)

ALGEBRA Find each measure.

5. \( m\angle N \)

6. \( m\angle L \)

7. \( m\angle C \)

8. \( m\angle A \)

9. \( m\angle J \)

10. \( m\angle K \)

11. \( m\angle S \)

12. \( m\angle R \)

13. ARENA A circus arena is lit by five lights equally spaced around the perimeter.

What is \( m\angle 1 \)?
14. FIELD OF VIEW The figure shows a top view of two people in front of a very tall rectangular wall. The wall makes a chord of a circle that passes through both people.

Which person has more of their horizontal field of vision blocked by the wall?

15. RHOMBI Paul is interested in circumscribing a circle around a rhombus that is not a square. He is having great difficulty doing so. Can you help him? Explain.

16. STREETS Three kilometers separate the intersections of Cross and Upton and Cross and Hope.

What is the distance between the intersection of Upton and Hope and the point midway between the intersections of Upton and Cross and Cross and Hope?

17. INSCRIBED HEXAGONS You will prove that the sum of the measures of alternate interior angles in an inscribed hexagon is 360.

a. How are \( \angle A \) and \( \angle BCF \) related? Similarly, how are \( \angle E \) and \( \angle DCF \) related?

b. Show that \( m \angle A + m \angle BCD + m \angle E = 360 \).
10.5 Warm Up

1. Find the value of x and y.

\[ (4x - 7)^\circ, (2x + 11)^\circ, (3y + 6)^\circ, (5y - 14)^\circ \]

2. Find the value of x

Common Core State Standards
G. CO. 12 Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.)
G.C.4 Construct a tangent line from a point outside a given circle to the circle.

Mathematical Practices
1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.

Learning Targets
- Students will be able to use properties of tangents.
- Students will be able to solve problems involving circumscribed polygons.

Section 10.5 Notes: Tangents

A _________________ is a line in the same plane as a circle that intersects the circle in exactly one point, called __________________________.

\[ \overline{AB} \] is tangent to \( \odot C \) at point \( A \). \( \overline{AB} \) and \( \overline{AB} \) are also called tangents.

A _________________ is a line, ray, or segment that is tangent to two circles in the same plane.

In each figure below, \( \ell \) is a common tangent of circles \( F \) and \( G \).

Example 1:

a) Using the figure to the right, draw the common tangents. If no common tangent exists, state no common tangent.

b) Using the figure to the right, draw the common tangents. If no common tangent exists, state no common tangent.
Example 2:

a) $KL$ is a radius of $\odot K$. Determine whether $LM$ is tangent to $\odot K$. Justify your answer.

b) $XY$ is a radius of $\odot X$. Determine whether $YZ$ is tangent to $\odot X$. Justify your answer.
Example 3:

a) In the figure, $\overline{WE}$ is tangent to $\odot D$ at $W$. Find the value of $x$.

b) In the figure, $\overline{IK}$ is tangent to $\odot J$ at $K$. Find the value of $x$.

Example 4:

a) $\overline{AC}$ and $\overline{BC}$ are tangent to $\odot Z$. Find the value of $x$.

b) $\overline{MN}$ and $\overline{MP}$ are tangent to $\odot Q$. Find the value of $x$. 
Circumscribed Polygons

A polygon is _____________________________ about a circle if every side of the polygon is tangent to the circle.

<table>
<thead>
<tr>
<th>Circumscribed Polygons</th>
<th>Polygons Not Circumscribed</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1" alt="Circumscribed Polygons" /></td>
<td><img src="image2" alt="Polygons Not Circumscribed" /></td>
</tr>
</tbody>
</table>

Example 5:

a) The round cookies are marketed in a triangular package to pique the consumer’s interest. If \( \triangle QRS \) is circumscribed about \( \odot T \), find the perimeter of \( \triangle QRS \).

![Image of \( \triangle QRS \)](image3)

b) A bouncy ball is marketed in a triangular package to pique the consumer’s interest. If \( \triangle ABC \) is circumscribed about \( \odot G \), find the perimeter of \( \triangle ABC \).

![Image of \( \triangle ABC \)](image4)

Challenge Problem

PQ is a tangent in circles R and S. Find PQ. Explain your reasoning.

![Image of circles R and S with PQ](image5)

Open Ended

<table>
<thead>
<tr>
<th>Draw a circumscribed triangle</th>
<th>Draw an inscribed triangle</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Learning Target Checklist

- Students will be able to recognize and use relationships between arcs and chords.
- Students will be able to recognize and use relationships between arcs, chords, and diameter.
**To construct the perpendicular bisector to $\overline{AC}$, put your compass point on $A$. Open the compass setting to more than half of $AC$. Draw an arc above and below $\overline{AC}$. Using the same compass setting, put the point on $C$. Draw an arc above and below $\overline{AC}$. The compass arcs should have intersected above and below $\overline{AC}$. Draw line $t$ through these intersecting arcs.**
Activity 1  Construct a Circle Inscribed in a Triangle

Step 1

Use

* Draw a triangle XYZ and construct two angle bisectors of the triangle to locate the incenter W.

Step 2

Construct a segment perpendicular to a side through the incenter. Label the intersection R.

Step 3

Set a compass of the length of WR. Put the point of the compass on W and draw a circle with that radius.

* To construct an angle bisector of ∠X, place the compass point on X and make an arc on sides XY and XZ. Using the same setting, place the point on each of the arcs and make a third and fourth arc that intersect far away from point X. Draw a line from this intersection point and X. This is an angle bisector.

Follow similar steps to construct the angle bisector of ∠Z. The point where the two angle bisectors intersect is the incenter. Label it W.

**To construct the perpendicular segment to XZ, put your compass point on W. Open the compass setting so that you can make 2 arcs on XZ. Use the same setting, place the compass point on one of the intersection points between an arc and XZ and make a third arc below XZ. Use the same setting and place the compass point on the other intersection point between an arc and XZ. Make a fourth arc below XZ that will intersect arc three. Label this intersection point R. Draw WR.
** Activity 2 ** Construct a Triangle Circumscribed About a Circle

* Draw this point on the circle.

** Using the same compass setting that you made your circle with, place the compass point on the point you made on the circle and make an arc. Then place the compass setting on the new arc and make a second arc. Place your compass setting on the second arc and make a third arc, etc. Continue this until you have a total of 6 arcs. One arc should be going through the original point.

*** To construct a perpendicular line to the first ray: Use a small compass setting and place your compass point on point A (as in diagram above). Make two arcs on the ray; one to the left of A and one to the right. Next, place your compass point on the arc to the right of point A and open the compass setting so that it is just pass point A. Draw an arc above and below point A. Use the same setting with your compass point on the arc to the left of point A, and draw another arc above and below point A. You should now have intersecting arcs above and below point A. Connect these intersecting points to create a perpendicular line to your ray.

Follow similar steps for the other two rays. Draw your perpendicular lines long enough so that they will intersect to form a triangle when you are finished.
Determine whether each segment is tangent to the given circle. Justify your answer.

1. $MP$

![Diagram of a circle with segments MP and additional labeled points](image)

2. $QR$

Find $x$. Assume that segments that appear to be tangent are tangent. Round to the nearest tenth if necessary.

3. $x$

![Diagram with labeled segments and expressions](image)

4. $x$

For each figure, find $x$. Then find the perimeter.

5. $x$

![Diagram of a triangle with segments and expressions](image)

6. $x$

![Diagram of a triangle with segments and expressions](image)

7. **CLOCKS** The design shown in the figure is that of a circular clock face inscribed in a triangular base. $AF$ and $FC$ are equal.
   a. Find $AB$.
   b. Find the perimeter of the clock.

![Diagram of a clock](image)

8. **CANALS** The concrete canal in Landtown is shaped like a “V” at the bottom. One day, Maureen accidentally dropped a cylindrical tube as she was walking and it rolled to the bottom of the dried out concrete canal. The figure shows a cross section of the tube at the bottom of the canal.

Compare the lengths $AV$ and $BV$. 

![Diagram of a canal with segments AV and BV](image)
9. PACKAGING  Taylor packed a sphere inside a cubic box. He had painted the sides of the box black before putting the sphere inside. When the sphere was later removed, he discovered that the black paint had not completely dried and there were black marks on the sides of the sphere at the points of tangency with the sides of the box. If the black marks are used as the vertices of a polygon, what kind of polygon results?

10. JEWELRY  Juanita is designing a pendant with a circular gem inscribed in a triangle. Find the values of $x$, $y$, and $z$. Then find the perimeter of the triangle.

11. ROLLING  A wheel is rolling down an incline. Twelve evenly spaced diameters form spokes of the wheel. When spoke 2 is vertical, which spoke will be perpendicular to the incline?

12. DESIGN  Amanda wants to make this design of circles inside an equilateral triangle.

a. What is the radius of the large circle to the nearest hundredth of an inch?

b. What are the radii of the smaller circles to the nearest hundredth of an inch?
1. Determine whether segment AB is tangent to the circle.

2. Assume that segments PW and QW are tangent to the circle. Find the value of x.

Common Core State Standards
G.C.4 Construct a tangent line from a point outside a given circle.

Mathematical Practices
3 Construct viable arguments and critique the reasoning of others.
1 Make sense of problems and persevere in solving them.

Section 10.6 Notes: Secants, Tangents, and Angle Measures

A ________ is a line that intersects a circle in exactly two points. Lines j and k are secants of \( \bigcirc C \).

Theorem 10.12

Words
If two secants or chords intersect in the interior of a circle, then the measure of an angle formed is one half the sum of the measure of the arcs intercepted by the angle and its vertical angle.

Example 1:
a) Find x.
b) Find $x$.

c) Find $x$.

Example 2:

a) Find $m \angle QPS$.

c) Find $m \angle FGI$.

b) Find $m \overline{BCD}$.

d) Find $m \overline{UVW}$.
**Example 3:**

What situation do we have for a: ______________________

a) Find $\overparen{BC}$.  

What situation do we have for c: ______________________

c) Find $\overparen{QS}$.  

What situation do we have for b: ______________________

b) Find $\overparen{XYZ}$.  

What situation do we have for d: ______________________

d) Find $\overparen{FIH}$.  

---

**Theorem 10.14**

**Words** If two secants, a secant and a tangent, or two tangents intersect in the exterior of a circle, then the measure of the angle formed is one half the difference of the measures of the intercepted arcs.
Example 4:

a) The diagram shows the path of a light ray as it hits a cut diamond. The ray is bent, or refracted, at points A, B, and C. If \( \angle \text{mAC} = 96^\circ \) and \( \angle \text{mS} = 35^\circ \), what is \( \angle \text{mRBT} \)?

b) The diagram shows the path of a light ray as it hits a cut diamond. The ray is bent, or refracted, at points X, Y, and W. If \( \angle \text{mXW} = 100^\circ \) and \( \angle \text{mT} = 30^\circ \), what is \( \angle \text{mVYU} \)?

Key Concept Review:

<table>
<thead>
<tr>
<th>Vertex of Angle is located…</th>
<th>Examples of this situation:</th>
<th>Angle Measurement formula:</th>
</tr>
</thead>
<tbody>
<tr>
<td>on the circle</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Inside the circle</td>
<td></td>
<td></td>
</tr>
<tr>
<td>outside the circle</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
New Vocabulary: Compare and contrast a tangent and a secant.

Challenge Problem
The circles below are concentric. What is $x$?

Learning Target Checklist
- I can find measures of angles formed by lines intersecting on or in a circle.
- I can find measures of angles formed by lines intersecting outside the circle.
Find each measure. Assume that segments that appear to be tangent are tangent.

1. \( m \angle 1 \)

2. \( m \angle 2 \)

3. \( m \angle 3 \)

4. \( m \angle 4 \)

5. \( m \angle 5 \)

6. \( m \angle 6 \)

7. \( m \angle R \)

8. \( m \angle K \)

9. \( m \angle U \)

10. \( m \angle S \)

11. \( m \overline{DP} \overline{A} \)

12. \( m \overline{LJ} \)

13. **TELESCOPES** Vanessa looked through her telescope at a mountainous landscape. The figure shows what she saw. Based on the view, approximately what angle does the side of the mountain that runs from A to B make with the horizontal?
14. **RADAR** Two airplanes were tracked on radar. They followed the paths shown in the figure.

What is the acute angle between their flight paths?

![Diagram of radar paths]

15. **EASELS** Francisco is a painter. He places a circular canvas on his A-frame easel and carefully centers it. The apex of the easel is 30° and the measure of arc $BC$ is 22°. What is the measure of arc $AB$?

![Diagram of easel]

16. **FLYING** When flying at an altitude of 5 miles, the lines of sight to the horizon looking north and south make about a 173.7° angle. How much of the longitude line directly under the plane is visible from 5 miles high?

![Diagram of longitude line and plane]

17. **STAINED GLASS** Pablo made the stained glass window shown. He used an inscribed square and equilateral triangle for the design.

   a. Label the angle measures on the outer edge of the triangle.

   ![Diagram of stained glass window]

   b. Label all of the arcs with their degree measure.
1. Find the $m\angle R$

2. Find the $m\overline{GJ}$

Common Core State Standards
G.C.4 Construct a tangent line from a point outside a given circle to the circle

Mathematical Practices
1 Make sense of problems and persevere in solving them
7 Look for and make use of structure

Learning Targets
- Students will be able to find measures of segments that intersect in the interior of a circle.
- Students will find measures of segments that intersect in the exterior of a circle.

Section 10.7 Notes: Special Segments in a Circle

When two chords intersect inside a circle, each chord is divided into two segments, called ________________.

**Theorem 10.15 Segments of Chords Theorem**

**Example** $AB \cdot BC = DB \cdot BE$

Example 1:

a) Find $x$.

b) Find $x$.

c) Find $x$.

d) Find $x$. 
Example 2:

a) BIOLOGY Biologists often examine organisms under microscopes. The circle represents the field of view under the microscope with a diameter of 2 mm. Determine the length of the organism if it is located 0.25 mm from the bottom of the field of view. Round to the nearest hundredth.

b) ARCHITECTURE Phil is installing a new window in an addition for a client’s home. The window is a rectangle with an arched top called an eyebrow. The diagram below shows the dimensions of the window. What is the radius of the circle containing the arc if the eyebrow portion of the window is not a semicircle?

A secant segment is a segment of a _______________ that has exactly one endpoint on the circle.

In the figure, \( \overline{AC}, \overline{AB}, \overline{AE} \) and \( \overline{AD} \) are secant segments.

A secant segment that lies in the exterior of the circle is called an _________________.

In the figure, \( \overline{AB} \) and \( \overline{AD} \) are external secant segments.

**Theorem 10.16 Secant Segments Theorem**

<table>
<thead>
<tr>
<th>Words</th>
<th>If two secants intersect in the exterior of a circle, then the product of the measures of one secant segment and its external secant segment is equal to the product of the measures of the other secant and its external secant segment.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Example</td>
<td></td>
</tr>
</tbody>
</table>
Example 3:

a) Find $x$.

![Diagram showing a triangle with a tangent segment $EH$ and a secant segment $EF$ intersecting outside the circle.]

b) Find $x$.

![Diagram showing a circle with a tangent segment $GI$ and a secant segment $GI$ intersecting outside the circle.]

A __________________________ is a segment of a tangent with one endpoint on the circle that is both the exterior and whole segment.

**Theorem 10.17**

**Words**

If a tangent and a secant intersect in the exterior of a circle, then the square of the measure of the tangent is equal to the product of the measures of the secant and its external secant segment.

**Example**

![Diagram illustrating the theorem with a tangent segment $JK$ and a secant segment $JM$.]

Example 4:

a) $LM$ is tangent to the circle. Find $x$. Round to the nearest tenth.

![Diagram showing a tangent segment $LM$ intersecting with a secant segment $LM$.]

b) Find $x$. Assume that segments that appear to be tangent are tangent.

![Diagram showing a tangent segment $MN$ and a secant segment $MO$.]

**New Vocabulary** Match each term with its definition by drawing a line to connect the two.

- **tangent segment**: a segment formed when two chords intersect inside a circle
- **secant segment**: a segment of a secant line that has exactly one endpoint on the circle
- **external secant segment**: a segment of a tangent line that has exactly one endpoint on the circle
- **chord segment**: a segment of a secant line that has an endpoint which lies in the exterior of the circle
Segments Intersecting Outside a Circle – Compare and contrast Theorem 10.16 with Theorem 10.15

<table>
<thead>
<tr>
<th>How are they the same?</th>
<th>How are they different?</th>
</tr>
</thead>
</table>

**Challenge Problem**

In the figure, a line tangent to circle M and a secant line intersect at R. Find \( a \). Show the steps that you used.

**Learning Target Checklist**

- Students will be able to find measures of segments that intersect in the interior of a circle.
- Students will find measures of segments that intersect in the exterior of a circle.
Find $x$ to the nearest tenth if necessary. Assume that segments that appear to be tangent are tangent.

1. 

2. 

3. 

4. 

5. 

6. 

7. 

8. 

9. 

10. **ICE SKATING** Ted skated through one of the face-off circles at a skating rink. His path through the circle is shown in the figure. Given that the face-off circle is 15 feet in diameter, what distance within the face-off circle did Ted travel?

11. **HORIZONS** Assume that Earth is a perfect sphere with a diameter of 7926 miles. From an altitude of $a$ miles, how long is the horizon line $h$?
12. AXLES The figure shows the cross-section of an axle held in place by a triangular sleeve. A brake extends from the apex of the triangle. When the brake is extended 2.5 inches into the sleeve, it comes into contact with the axle. What is the diameter of the axle?

13. ARCHEOLOGY Scientists unearthed part of a circular wall. They made the measurements shown in the figure. Based on the information in the figure, what was the radius of the circle?

14. PIZZA DELIVERY Pizza Power is located at the intersection of Northern Boulevard and Highway 1 in a city with a circular highway running all the way around its outskirts. The radius of the circular highway is 13 miles. Pizza Power puts the map shown below on its take-out menus.

a. How many miles away is the Circular Highway from Pizza Power if you travel north on Highway 1?

b. The city builds a new road along the diameter of Circular Highway that passes through the intersection of Northern Boulevard and Highway 1. Along this new road, about how many miles is it (the shorter way) to the Circular Highway from Pizza Power?
10.8 Warm Up

1. Find the value of x.  
2. Find the value of x.

Common Core State Standards
G.GPE.1 Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of circle given by an equation. 
G. GPE. 6 Find the point on a directed line segment between the given points that partitions the segment in a given ratio.

Mathematical Practices
2 Reason abstractly and quantitatively
7 Look for and make sure of structure

Learning Targets
• Students will be able to write the equation of a circle.
• Students will be able to graph a circle on the coordinate plane.

Section 10.8 Notes: Equations of Circles

Key Concept: Equation of a Circle in Standard Form

The standard form of the equation of a circle with center at \((h, k)\) and radius \(r\) is \((x - h)^2 + (y - k)^2 = r^2\)

The standard form of the equation of a circle is also called the center-radius form.

Example 1:

a) Write the equation of the circle with a center at \((3, -3)\) and a radius of 6.

b) Write the equation of the circle graphed below.

c) Write the equation of the circle graphed below.
Example 2:

a) Write the equation of the circle that has its center at \((-3, -2)\) and passes through \((1, -2)\).

b) Write the equation of the circle that has its center at \((-1, 0)\) and passes through \((3, 0)\).

Example 3:

a) The equation of a circle is \(x^2 - 4x + y^2 + 6y = -9\). State the coordinates of the center and the measure of the radius. Then graph the equation.

b) Which of the following is the graph of \(x^2 + y^2 - 10y = 0\)?

Example 4:

a) ELECTRICITY  Strategically located substations are extremely important in the transmission and distribution of a power company’s electric supply. Suppose three substations are modeled by the points D(3, 6), E(−1, 1), and F(3, −4). Determine the location of a town equidistant from all three substations, and write an equation for the circle.

b) AMUSEMENT PARKS  The designer of an amusement park wants to place a food court equidistant from the roller coaster located at (4, 1), the Ferris wheel located at (0, 1), and the boat ride located at (4, −3). Determine the location for the food court.
Example 5:

a) Find the point(s) of intersection between \(x^2 + y^2 = 32\) and \(y = x + 8\).

b) Find the points of intersection between \(x^2 + y^2 = 16\) and \(y = -x\).

**Graph the circle given by the equation**

\[(x + 4)^2 + (y - 2)^2 = 16.\]

**Rewrite the equation in standard form.**

______________________________

**Identify the center. _____**

**Identify the radius. _____**

**Use the center and radius to identify four points on the circle.**

______________________________

______________________________

**Challenge Problem**

A circle has the equation \((x - 5)^2 + (y + 7)^2 = 16\). If the center of the circle is shifted 3 units right and 9 units up, what will be the equation of the new circle? Explain your reasoning.

**Learning Target Checklist**

- [ ] Students will be able to write the equation of a circle.
- [ ] Students will be able to graph a circle on the coordinate plane.
Write the equation of each circle.

1. center at (0, 0), diameter 18

2. center at (–7, 11), radius 8

3. center at (–1, 8), passes through (9, 3)

4. center at (–3, –3), passes through (–2, 3)

For each circle with the given equation, state the coordinates of the center and the measure of the radius. Then graph the equation.

5. \(x^2 + y^2 = 4\)

6. \(x^2 + y^2 + 6x - 6y + 9 = 0\)

Write an equation of a circle that contains each set of points. Then graph the circle.

7. \(A(–2, 2), B(2, –2), C(6, 2)\)

8. \(R(5, 0), S(–5, 0), T(0, –5)\)

Find the point(s) of intersection, if any, between each circle and line with the equations given.

9. \(x^2 + y^2 = 25; y = x\)

10. \((x + 4)^2 + (y - 3)^2 = 25; y = x + 2\)

11. **EARTHQUAKES** When an earthquake strikes, it releases seismic waves that travel in concentric circles from the epicenter of the earthquake. Seismograph stations monitor seismic activity and record the intensity and duration of earthquakes. Suppose a station determines that the epicenter of an earthquake is located about 50 kilometers from the station. If the station is located at the origin, write an equation for the circle that represents one of the concentric circles of seismic waves of the earthquake.
12. DESIGN Arthur wants to write the equation of a circle that is inscribed in the square shown in the graph.

What is the equation of the desired circle?

13. DRAFTING The design for a park is drawn on a coordinate graph. The perimeter of the park is modeled by the equation \((x - 3)^2 + (x - 7)^2 = 225\). Each unit on the graph represents 10 feet. What is the radius of the actual park?

14. WALLPAPER The design of a piece of wallpaper consists of circles that can be modeled by the equation \((x - a)^2 + (y - b)^2 = 4\), for all even integers \(b\). Sketch part of the wallpaper on a grid.

15. SECURITY RING A circular safety ring surrounds a top-secret laboratory. On one map of the laboratory grounds, the safety ring is given by the equation \((x - 8)^2 + (y + 2)^2 = 324\). Each unit on the map represents 1 mile. What is the radius of the safety ring?

16. DISTANCE Cleo lives the same distance from the library, the post office, and her school. The table below gives the coordinates of these places on a map with a coordinate grid where one unit represents one yard.

<table>
<thead>
<tr>
<th>Location</th>
<th>Coordinates</th>
</tr>
</thead>
<tbody>
<tr>
<td>Library</td>
<td>(–78, 202)</td>
</tr>
<tr>
<td>Post Office</td>
<td>(111, 193)</td>
</tr>
<tr>
<td>School</td>
<td>(202, –106)</td>
</tr>
</tbody>
</table>

a. What are the coordinates of Cleo’s home? Sketch the circle on a map locating all three places and Cleo’s home.

b. How far is Cleo’s house from the places mentioned?

c. Write an equation for the circle that passes through the library, post office, and school.
Warm Up 11.3

1. Use Circle D to find the length of each arc. Round to the nearest hundredth.
   a. \( \overline{LM} \) if the radius is 5 inches.
   b. \( \overline{MN} \) if the diameter is 3 yards.
   c. \( \overline{KL} \) if \( JD = 7 \) centimeters
   d. \( \overline{JK} \) if \( NL = 12 \) feet.

Common Core State Standards

G. C. 5 Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.

G.GMD.1 Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Attend to precision

Learning Targets

- Students will be able to find areas of circles.
- Students will find areas of sectors of circles.

Section 11.3 Notes: Areas of Circles and Sectors

In Lesson 10-1, you learned that the formula for the circumference \( C \) of a circle with radius \( r \) is given by \( C = 2\pi r \). You can use this formula to develop the formula for the area of a circle.

Below, a circle with radius \( r \) and circumference \( C \) has been divided into congruent pieces and then rearranged to form a figure that resembles a parallelogram.

As the number of congruent pieces increases, the rearranged figure more closely approaches a parallelogram. The base of the parallelogram is \( \frac{1}{2}C \) and the height is \( r \), so its area is \( \frac{1}{2}C \cdot r \). Since \( C = 2\pi r \), the area of the parallelogram is also \( \frac{1}{2}(2\pi r)r \) or \( \pi r^2 \).

Example 1: An outdoor accessories company manufactures circular covers for outdoor umbrellas. If the cover is 8 inches longer than the umbrella on each side, find the area of the cover in square inches.

Example 2: Find the radius of a circle with an area of 58 square inches.
A slice of a circular pizza is an example of a sector of a circle. A ________ is a region of a circle bounded by a central angle and its intercepted major or minor arc. The formula for the area of a sector is similar to the formula for arc length.

**Key Concept Area of a Sector**

The ratio of the area $A$ of a sector to the area of the whole circle, $\pi r^2$, is equal to the ratio of the degree measure of the intercepted arc $x$ to 360.

$$A = \frac{x}{360} \cdot \pi r^2$$

---

**Example 3:**

a) A pie has a diameter of 9 inches and is cut into 10 congruent slices. What is the area of one slice to the nearest hundredth?

b) A pizza has a diameter of 14 inches and is cut into 8 congruent slices. What is the area of one slice to the nearest hundredth?

---

**Area of a Sector Practice:** Find the area of the shaded sector. Round to the nearest tenth.

a)  

b)  

---

**Helping You Remember:** A good way to remember something is to explain it to someone else. Suppose Jimmy is having trouble remembering which formula is for circumference and which is for area. How can you help him out?

---

**Challenge Problem:** Find the area of the shaded region. Round to the nearest tenth.

---

**Learning Target Checklist**

- Students will be able to find areas of circles.
- Students will find areas of sectors of circles.
Geometry
Section 11.3 Worksheet

Find the area of each circle. Round to the nearest tenth.

1. 2. 3.

Find the indicated measure. Round to the nearest tenth.

4. The area of a circle is 3.14 square centimeters. Find the diameter.

5. Find the diameter of a circle with an area of 855.3 square millimeters.

6. The area of a circle is 201.1 square inches. Find the radius.

7. Find the radius of a circle with an area of 2290.2 square feet.

Find the area of each shaded sector. Round to the nearest tenth.

8. 9. 10.

11. CLOCK Sadie wants to draw a clock face on a circular piece of cardboard. If the clock face has a diameter of 20 centimeters and is divided into congruent pieces so that each sector is 30°, what is the area of each piece?
12. LOBBY The lobby of a bank features a large marble circular table. The diameter of the circle is 15 feet. What is the area of the circular table? Round your answer to the nearest tenth.

13. PORTHOLES A circular window on a ship has a radius of 8 inches. What is the area of the window? Round your answer to the nearest hundredth.

14. PEACE SYMBOL The symbol below, a circle separated into 3 equal sectors, has come to symbolize peace. Suppose the circle has radius $r$. What is the area of each sector?

15. SOUP CAN Julie needs to cover the top and bottom of a can of soup with construction paper to include in her art project. Each circle has a diameter of 7.5 centimeters. What is the total area of the can that Julie must cover?

16. POOL A circular pool is surrounded by a circular sidewalk. The circular sidewalk is 3 feet wide. The diameter of the sidewalk and pool is 26 feet.

   a. What is the diameter of the pool?

   b. What is the area of the sidewalk and pool?

   c. What is the area of the pool?
11.4 Warm Up

1. Which of the objects shown below could be sliced to create a circular cross-section?

*Please note that the height of the cylinder is greater than the diameter of its base, the height of the cone is greater than the diameter of its base, and the length of the prism is greater than any side of its equilateral base.

![Objects](image)

Common Core State Standards

G.MG.3 Apply geometric methods to solve problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).

Mathematical Practices

1. Make sense of problems and persevere in solving them.
6. Attend to precision.

Learning Targets

- Students will be able to find areas of regular polygons.
- Find areas of composite figures.

Section 11.4 Notes: Areas of Regular Polygons and Composite Figures

In the figure, a regular pentagon is inscribed in circle P, and circle P is circumscribed about the pentagon. The _____________ and the ________________ are also the center and the radius of its circumscribed circle.

A segment drawn from the center of a regular polygon perpendicular to a side of the polygon is called an ________________. Its length is the height of an isosceles triangle that has two radii as legs.

A __________________________ has its vertex at the center of the polygon and its sides pass through consecutive vertices of the polygon. The measure of each central angle of a regular n-gon is \( \frac{360}{n} \).

Example 1: In the figure, pentagon \( PQRST \) is inscribed in circle X. Identify the center, a radius, an apothem, and a central angle of the polygon. Then find the measure of a central angle.

Example 2: The top of the table shown is a regular hexagon with a side length of 3 feet and an apothem of 1.7 feet. What is the area of the tabletop to the nearest tenth?
From Example 2, we can develop a formula for the area of a regular $n$-gon with side length $s$ and apothem $a$.

### KeyConcept: Area of a Regular Polygon

**Words:** The area $A$ of a regular $n$-gon with side length $s$ is one half the product of the apothem $a$ and perimeter $P$.

**Symbols:**

---

**Example 3:**

a) Find the area of the regular hexagon. Round to the nearest tenth.

![Hexagon](image1.png)

b) Find the area of the regular pentagon. Round to the nearest tenth.

![Pentagon](image2.png)

---

A composite figure is a figure that can be separated into regions that are basic figures, such as triangles, rectangles, trapezoids, and circles. To find the area of a composite figure, find the area of each basic figure and then use the Area Addition Postulate.

**Example 4:**

a) The dimensions of an irregularly shaped pool are shown. What is the area of the surface of the pool?

![Pool 1](image3.png)

b) Find the area of the figure in square feet. Round to the nearest tenth if necessary.

![Pool 2](image4.png)
Example 5:
a) Find the area of the shaded figure.

b) Cara wants to wallpaper one wall of her family room. She has a fireplace in the center of the wall. Find the area of the wall around the fireplace.

Identify the parts of the regular polygon.

- What is the center?
- Name a radius.
- Name a central angle.
- Name an apothem.

Challenge Problem
JoAnn wants to lay 12” x 12” tile on her bathroom floor.

a. Find the area of the bathroom floor in her apartment floor plan.
b. If the tile comes in boxes of 15 and JoAnn buys no extra tiles, how many boxes will she need?

Learning Checklist
- Students will be able to find areas of regular polygons.
- Find areas of composite figures.
Find the area of each regular polygon. Round to the nearest tenth.

1. [Diagram of a triangle with 14 cm side]

2. [Diagram of a pentagon with 7 m side]

Find the area of each figure. Round to the nearest tenth if necessary.

3. [Diagram of a figure with 20 mm and 20 mm sides]

4. [Diagram of a figure with 38 ft and 22 ft sides]

5. [Diagram of a figure with 9 m and 7 ft sides]

6. [Diagram of a figure with 13 in. and 30 in. sides]

7. **LANDSCAPING** One of the displays at a botanical garden is a koi pond with a walkway around it. The figure shows the dimensions of the pond and the walkway.

   a. Find the area of the pond to the nearest tenth.

   b. Find the area of the walkway to the nearest tenth.
8. **YIN-YANG SYMBOL** A well-known symbol from Chinese culture is the yin-yang symbol, shown below. Suppose the large circle has radius \( r \), the small circles have radius \( \frac{r}{8} \), and the S-curve is two semicircles, each with radius \( \frac{r}{2} \). In terms of \( r \), what is the area of the black region?

![Yin-Yang Symbol](image)

9. **PYRAMIDS** Martha’s clubhouse is shaped like a square pyramid with four congruent equilateral triangles for its sides. All of the edges are 6 feet long. What is the total surface area of the clubhouse including the floor? Round your answer to the nearest hundredth.

10. **MINIATURE GOLF** The plan for a miniature golf hole is shown below. The right angle in the drawing is a central angle. What is the area of the playing surface? Round your answer to the nearest hundredth of a square meter.

![Miniature Golf Hole](image)

11. **TRACK** A running track has an inner and outer edge. Both the inner and outer edges consist of two semicircles joined by two straight line segments. The straight line segments are 100 yards long. The radii of the inner edge semicircles are 25 yards each and the radii of the outer edge semicircles are 32 yards each. What is the area of the track? Round your answer to the nearest hundredth of a yard.

![Running Track](image)

12. **SEMICIRCLES** Bridget arranged three semicircles in the pattern shown. The right triangle has side lengths 6, 8, and 10 inches.

   a. What is the total area of the three semicircles? Round your answer to the nearest hundredth of a square inch.

   ![Semicircles and Triangle](image)

   b. If the right triangle had side lengths \( \sqrt{21}, \sqrt{79} \), and 10 inches, what would the total area of the three semicircles be? Round your answer to the nearest hundredth of a square inch.
Common Core State Standards
G.GMD.4 Identify the shapes of two-dimensional cross-sections of three dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.

Mathematical Practices
5 Use appropriate tools strategically.
1 Make sense of problems and persevere in solving them.

Learning Targets
- Students will be able to draw isometric views of three-dimensional figures.
- Students will be able to investigate cross-sections of three-dimensional figures.

Section 12.1 Notes: Representations of Three-dimensional Figures

Example 1
BAKERY A customer ordered a two-layer sheet cake. Determine the shape of each cross section of the cake below.

Example 2
Determine the shape of the cross section shown.

Name the different cross sections of a cylinder. State how the plane would have to intersect the cylinder to get that particular cross section.

1. __________________________
   __________________________

2. __________________________
   __________________________

3. __________________________
   __________________________
**Challenge Problems**
The figure at the right is a cross section of a geometric solid. Describe a solid and how the cross section was made.

![Cross section](image)

Draw the top view, front view, and left view of the solid figure shown below.

![Solid figure](image)

**Learning Target Checklist**

- [ ] Students will be able to draw isometric views of three-dimensional figures.
- [ ] Students will be able to investigate cross-sections of three-dimensional figures.
Describe each cross section.

1.

2.

3.

4.

5. **LABELS** Jamal removes the label from a cylindrical soup can to earn points for his school. Sketch the shape of the label.

6. **BLOCKS** Margo’s three-year-old son made the magnetic block sculpture shown below in corner view.

   ![Corner View](image)

   Draw the right view of the sculpture.

7. **CUBES** Nathan marks the midpoints of three edges of a cube as shown. He then slices the cube along a plane that contains these three points. Describe the resulting cross section.
8. **ENGINEERING** Stephanie needs an object whose top view is a circle and whose left and front views are squares. Describe an object that will satisfy these conditions.

9. **DESK SUPPORTS** The figure shows the support for a desk.

   a. Draw the top view.

   b. Draw the front view.

   c. Draw the right view.
12.2 Warm Up

1. Find the area of the figure. Round to the nearest tenth.

![Figure 1]

2. Find the area of the figure. Round to the nearest tenth.

![Figure 2]

Common Core State Standards

G.MG.3 Apply geometric methods to solve problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Attend to precision.

Learning Targets

- Students will be able to find lateral areas and surface areas of prisms.
- Students will be able to find lateral areas and surface areas of cylinders.

Section 12.2 Notes: Surface Areas of Prisms and Cylinders

In a solid figure, faces that are not bases are called __________________. Lateral faces intersect each other at the ________________, which are all parallel and congruent. The lateral faces intersect the base at the ________________. The ________________ is a perpendicular segment that joins the planes of the bases. The height is the __________ of the altitude.

Recall that a prism is a polyhedron with two parallel congruent bases

The lateral area $L$ of a prism is the sum of the areas of the lateral faces. The net at the right shows how to find the lateral area of a prism.

$$L = a(h) + b(h) + c(h)$$

Sum of areas of lateral faces

$$(a + b + c)h$$

Distributive Property

$$Ph$$

$P = a + b + c$

From this point on, you can assume that solids in the text are right solids. If a solid is oblique, it will be clearly stated.
Example 1: Lateral Area of a Prism Find the lateral area of the prism. Round your answers to the nearest hundredth.

The surface area of a prism is the sum of the lateral area and the areas of the bases.

![Key Concept: Surface Area of a Prism]

**Words**
The surface area $S$ of a right prism is $S = L + 2B$, where $L$ is its lateral area and $B$ is the area of a base.

**Symbols**

Example 2: Find the surface area of the rectangular prism.

Example 3: Find the Surface Area of the regular hexagonal prism.

![Key Concept: Areas of a Cylinder]

**Words**
The lateral area $L$ of a right cylinder is $L = 2\pi rh$, where $r$ is the radius of a base and $h$ is the height.

The surface area $S$ of a right cylinder is $S = 2\pi rh + 2\pi r^2$, where $r$ is the radius of a base and $h$ is the height.

**Symbols**
Example 4: Find the lateral area and the surface area of the cylinder. Round to the nearest thousandth.

Example 5: Real-World Application
A can of soup is covered with the label shown. What is the radius of the soup can?

New Vocabulary – Label the elements of this oblique pentagonal prism with the correct terms.

Challenge Problem
A right prism has a height of $h$ units and a base that is an equilateral triangle of side $l$ units. Find the general formula for the total surface area of the prism. Explain your reasoning.
Find the lateral area and surface area of each prism. Round to the nearest tenth if necessary.

1. [Diagram of a triangular prism]
   
   Lateral Area__________________
   Surface Area__________________

2. [Diagram of a rectangular prism]
   
   Lateral Area__________________
   Surface Area__________________

3. [Diagram of a triangular prism with unspecified dimensions]
   
   Lateral Area__________________
   Surface Area__________________

4. [Diagram of a rectangular prism with unspecified dimensions]
   
   Lateral Area__________________
   Surface Area__________________

5. [Diagram of a rectangular prism with unspecified dimensions]
   
   Lateral Area__________________
   Surface Area__________________

6. [Diagram of a rectangular prism with unspecified dimensions]
   
   Lateral Area__________________
   Surface Area__________________
Surface Areas of Prisms and Cylinders

Find the lateral and surface area of each prism. Round to the nearest tenth if necessary.

1. [Diagram of a prism with dimensions 15 cm, 32 cm, 15 cm]
2. [Diagram of a prism with dimensions 10 ft, 5 ft, 8 ft]
3. [Diagram of a prism with dimensions 2 m, 11 m, 2 m]
4. [Diagram of a prism with dimensions 4 yd, 9.5 yd, 4 yd]

Find the lateral area and surface area of each cylinder. Round to the nearest tenth.

5. [Diagram of a cylinder with dimensions 5 ft, 7 ft]
6. [Diagram of a cylinder with dimensions 4 m, 8.5 m]
7. [Diagram of a cylinder with dimensions 19 in, 17 in]
8. [Diagram of a cylinder with dimensions 12 m, 30 m]

9. LOGOS The Z company specializes in caring for zebras. They want to make a 3-dimensional “Z” to put in front of their company headquarters. The “Z” is 15 inches thick and the perimeter of the base is 390 inches.

What is the lateral surface area of this “Z”?
10. STAIRWELLS Management decides to enclose stairs connecting the first and second floors of a parking garage in a stairwell shaped like an oblique rectangular prism.

What is the lateral surface area of the stairwell?

11. CAKES A cake is a rectangular prism with height 4 inches and base 12 inches by 15 inches. Wallace wants to apply frosting to the sides and the top of the cake. What is the surface area of the part of the cake that will have frosting?

12. EXHAUST PIPES An exhaust pipe is shaped like a cylinder with a height of 50 inches and a radius of 2 inches. What is the lateral surface area of the exhaust pipe? Round your answer to the nearest hundredth.

13. TOWERS A circular tower is made by placing one cylinder on top of another. Both cylinders have a height of 18 inches. The top cylinder has a radius of 18 inches and the bottom cylinder has a radius of 36 inches.

a. What is the total surface area of the tower? Round your answer to the nearest hundredth.

b. Another tower is constructed by placing the original tower on top of another cylinder with a height of 18 inches and a radius of 54 inches. What is the total surface area of the new tower? Round your answer to the nearest hundredth.
12.3 Warm Up

1. Find the surface area of the prism. Round to the nearest tenth.

2. Find the surface area of the cylinder. Round to the nearest tenth.

Common Core State Standards
G.MG.1 Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree truck or a human torso as a cylinder).

Mathematical Practices
1 Make sense of problems and persevere in solving them.
6 Attend to precision.

Learning Targets
- Students will be able to find lateral areas and surface areas of pyramids.
- Students will be able to find lateral areas and surface areas of cones.

Section 12.3 Notes: Surface Areas of Pyramids and Cones

Lateral Area and Surface Area of Pyramids
The ____________________ of a pyramid intersect at a common point called the ______________. Two lateral faces intersect at a ______________. A lateral face and the base intersect at a ______________. The ______________ is the segment from the vertex perpendicular to the base.

A ______________ has a base that is a regular polygon and the altitude has an endpoint at the center of the base. All the lateral edges are congruent and all the lateral faces are congruent isosceles triangles. The height of each lateral face is called the __________, ℓ, of a pyramid.

Example 1: Find the lateral area of the square pyramid to the nearest tenth.
Example 2: Find the surface area of the square pyramid to the nearest tenth.

Example 3: Find the surface area of the regular pyramid. Round to the nearest hundredth.

Lateral Area and Surface Area of Cones

Recall that a cone has a circular base and a vertex. The axis of a cone is the segment with endpoints at the vertex and the center of the base. If the axis is also the altitude, then the cone is a _________. If the axis is not the altitude, then the cone is an ___________.

Example 4: Find the surface area of the cone. Round to the nearest tenth.
Using the solid and model provided, write in the formulas for each solid.

<table>
<thead>
<tr>
<th>Solid</th>
<th>Model</th>
<th>Lateral Area</th>
<th>Surface Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prism</td>
<td><img src="image" alt="Prism" /></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cylinder</td>
<td><img src="image" alt="Cylinder" /></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pyramid</td>
<td><img src="image" alt="Pyramid" /></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cone</td>
<td><img src="image" alt="Cone" /></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Challenge Problems**
Determine whether the following statement is true or false. Explain your reasoning.

*A regular polygonal pyramid and a cone both have height $h$ units and base perimeter $P$ units. Therefore, they have the same total surface area.*

Classify the following statement as always, sometimes, or never true. Justify your reasoning.

*The surface area of a cone of radius $r$ and height $h$ is less than the surface area of a cylinder of radius $r$ and height $h.*

**Learning Target Checklist**
- Students will be able to find lateral areas and surface areas of pyramids.
- Students will be able to find lateral areas and surface areas of cones.
Geometry
12.3 Worksheet

Find the lateral area and surface area of each regular pyramid. Round to the nearest tenth if necessary.

1. 

![Diagram of a pyramid with dimensions 10 yd and 9 yd]

2. 

![Diagram of a pyramid with dimensions 12 m and 7 m]

3. 

![Diagram of a pyramid with dimensions 13 ft and 5 ft]

4. 

![Diagram of a pyramid with dimensions 8 cm and 2.5 cm]

Find the lateral area and surface area of each cone. Round to the nearest tenth if necessary.

5. 

![Diagram of a cone with dimensions 5 m and 4 m]

6. 

![Diagram of a cone with dimensions 7 cm and 21 cm]

7. Find the surface area of a cone if the height is 14 centimeters and the slant height is 16.4 centimeters.

8. Find the surface area of a cone if the height is 12 inches and the diameter is 27 inches.

9. **GAZEBOS** The roof of a gazebo is a regular octagonal pyramid. If the base of the pyramid has sides of 0.5 meter and the slant height of the roof is 1.9 meters, find the area of the roof.

10. **HATS** Wendy bought a conical hat on a recent trip to central Vietnam. The basic frame of the hat is 16 hoops of bamboo that gradually diminish in size. The hat is covered in palm leaves. If the hat has a diameter of 50 centimeters and a slant height of 32 centimeters, what is the lateral area of the conical hat?

11. **PAPER MODELS** Patrick is making a paper model of a castle. Part of the model involves cutting out the net shown and folding it into a pyramid. The pyramid has a square base. What is the lateral surface area of the resulting pyramid?
12. **TETRAHEDRON** Sung Li builds a paper model of a regular tetrahedron, a pyramid with an equilateral triangle for the base and three equilateral triangles for the lateral faces. One of the faces of the tetrahedron has an area of 17 square inches. What is the total surface area of the tetrahedron?

13. **PAPERWEIGHTS** Daphne uses a paperweight shaped like a pyramid with a regular hexagon for a base. The side length of the regular hexagon is 1 inch. The altitude of the pyramid is 2 inches.

What is the lateral surface area of this pyramid? Round your answers to the nearest hundredth
12.4 Warm Up

1. Find the surface area of the cone. Round to the nearest tenth.

2. Find the surface area of the regular pyramid. Round to the nearest tenth.

Common Core State Standards
G.GMD.1 Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone.
G.GMD.3 Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.

Mathematical Practices
1. Make sense of problems and persevere in solving them.
7. Look for and make use of structure.

Learning Targets
- Students will be able to find volumes of prisms.
- Students will be able to find volumes of cylinders.

Section 12.4 Notes: Volumes of Prisms and Cylinders

KeyConcept Volume of a Prism

<table>
<thead>
<tr>
<th>Words</th>
<th>The volume $V$ of a prism is $V = Bh$, where $B$ is the area of a base and $h$ is the height of the prism.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Symbols</td>
<td></td>
</tr>
</tbody>
</table>

Example 1: Find the volume of the prism. Make sure to label the correct units.

Critique Francisco and Valerie each calculated the volume of an equilateral triangular prism with an apothem of 4 units and height of 5 units. Is either of them correct? Explain your reasoning.

**Francisco**

$V = Bh$

$= \frac{1}{2} aP \cdot h$

$= \frac{1}{2}(4)(24\sqrt{3}) \cdot 5$

$= 240\sqrt{3}$ cubic units

**Valerie**

$V = Bh$

$= \frac{\sqrt{3}}{2} s^2 \cdot h$

$= \frac{\sqrt{3}}{2} (4\sqrt{3})^2 \cdot 5$

$= 120\sqrt{3}$ cubic units
Example 2: Find the volume of the cylinder. Make sure to label the correct units and leave your answer in terms of $\pi$.

Example 3: Find the volume of the oblique cylinder. Label your units and leave your answer in terms of $\pi$. 
**Challenge Problem**

The cylindrical can below is used to fill a container with liquid. It takes three full cans to fill the container. Describe possible dimensions of the container if it is each of the following shapes.

a. Rectangular prism  
b. Square prism  
c. Triangular prism with a right triangle as the base.

![Cylindrical can with dimensions 2 in. and 5 in.]

**Learning Target Checklist**

- Students will be able to find volumes of prisms.
- Students will be able to find volumes of cylinders.
Geometry
12.4 Worksheet

Find the volume of each prism or cylinder. Round to the nearest tenth if necessary.

1. \( V = \)__________

2. \( V = \)__________

3. \( V = \)__________

4. \( V = \)__________

5. \( V = \)__________

6. \( V = \)__________

7. **AQUARIUM** Mr. Gutierrez purchased a cylindrical aquarium for his office. The aquarium has a height of \(25 \frac{1}{2}\) inches and a radius of 21 inches.

   a. What is the volume of the aquarium in cubic feet?

   b. If there are 7.48 gallons in a cubic foot, how many gallons of water does the aquarium hold?

   c. If a cubic foot of water weighs about 62.4 pounds, what is the weight of the water in the aquarium to the nearest five pounds?

8. **TRASH CANS** The Meyer family uses a kitchen trash can shaped like a cylinder. It has a height of 18 inches and a base diameter of 12 inches. What is the volume of the trash can? Round your answer to the nearest tenth of a cubic inch.

9. **BENCH** Inside a lobby, there is a piece of furniture for sitting. The furniture is shaped like a simple block with a square base 6 feet on each side and a height of \(1 \frac{3}{5}\) feet.

   What is the volume of the seat?
10. FRAMES Margaret makes a square frame out of four pieces of wood. Each piece of wood is a rectangular prism with a length of 40 centimeters, a height of 4 centimeters, and a depth of 6 centimeters. What is the total volume of the wood used in the frame?

11. PENCIL GRIPS A pencil grip is shaped like a triangular prism with a cylinder removed from the middle. The base of the prism is a right isosceles triangle with leg lengths of 2 centimeters. The diameter of the base of the removed cylinder is 1 centimeter. The heights of the prism and the cylinder are the same, and equal to 4 centimeters.

What is the exact volume of the pencil grip?
12.5 Warm Up

1. Find the volume of the cylinder. Round to the nearest tenth.

```
6 yd

10 yd
```

2. Find the volume of the oblique prism. Round to the nearest tenth.

```
4 cm

17 cm

18 cm
```

Common Core State Standards
G.GMD.1 Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone.
G.GMD.3 Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.

Mathematical Practices
1 Make sense of problems and persevere in solving them.
7 Look for and make use of structure.

Learning Targets
- Students will be able to find volumes of pyramids.
- Students will be able to find volumes of cones.

Section 12.5 Notes: Volumes of Pyramids and Cones

**Key Concept** Volume of a Pyramid

<table>
<thead>
<tr>
<th>Words</th>
<th>The volume of a pyramid is ( V = \frac{1}{3}Bh ), where ( B ) is the area of the base and ( h ) is the height of the pyramid.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Symbols</td>
<td></td>
</tr>
</tbody>
</table>

**Example 1:** Find the volume of the square pyramid. Label the units.

```
7 in.

3 in.
```

**Key Concept** Volume of a Cone

<table>
<thead>
<tr>
<th>Words</th>
<th>The volume of a circular cone is ( V = \frac{1}{3}Bh ), or ( V = \frac{1}{3}\pi r^2 h ), where ( B ) is the area of the base, ( h ) is the height of the cone, and ( r ) is the radius of the base.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Symbols</td>
<td></td>
</tr>
</tbody>
</table>
Example 2: Find the volume of the cone. Leave your answers in terms of $\pi$ and label the units.

Example 3: Find the volume of the oblique cone. Round your answers to the tenth.

Example 4: Real-world application. SCULPTURE

At the top of a stone tower is a pyramidion in the shape of a square pyramid. This pyramid has a height of 52.5 centimeters and the base edges are 36 centimeters. What is the volume of the pyramidion? Round to the nearest tenth.

In the below chart, fill out the volume formulas for each individual solid.

<table>
<thead>
<tr>
<th>Solid</th>
<th>Prism</th>
<th>Cylinder</th>
<th>Pyramid</th>
<th>Cone</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model</td>
<td><img src="image1.png" alt="Model" /></td>
<td><img src="image2.png" alt="Model" /></td>
<td><img src="image3.png" alt="Model" /></td>
<td><img src="image4.png" alt="Model" /></td>
</tr>
<tr>
<td>Volume</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Helping you remember: Many students find it easier to remember mathematical formulas if they can put them in words. Use words to describe in one sentence how to find the volume of any pyramid or cone.

Learning Target Checklist

- [ ] Students will be able to find volumes of pyramids.
- [ ] Students will be able to find volumes of cones.

Challenge Problem

Alexandra and Cornelio are calculating the volume of the cone at the right. Is either of them correct? Explain your answer.

**Alexandra**

\[ V = \frac{1}{3} Bh \]

\[ = \frac{1}{3} \pi (5^2)(15) \]

\[ \approx 340.3 \text{ cm}^3 \]

**Cornelio**

\[ s^2 + 12^2 = t^2 \]

\[ \sqrt{s^2 + 12^2} = t \]

\[ V = \frac{1}{3} Bh \]

\[ = \frac{1}{3} \pi (5^2)(12) \]

\[ \approx 314.2 \text{ cm}^3 \]
Find the volume of each pyramid or cone. Round to the nearest tenth if necessary.

1. \( V = \) ________

2. \( V = \) ________

3. \( V = \) ________

4. \( V = \) ________

5. \( V = \) ________

6. \( V = \) ________

7. **CONSTRUCTION** Mr. Ganty built a conical storage shed. The base of the shed is 4 meters in diameter and the height of the shed is 3.8 meters. What is the volume of the shed?

8. **HISTORY** The start of the pyramid age began with King Zoser’s pyramid, erected in the 27th century B.C. In its original state, it stood 62 meters high with a rectangular base that measured 140 meters by 118 meters. Find the volume of the original pyramid.

9. **SCULPTING** A sculptor wants to remove stone from a cylindrical block 3 feet high and turn it into a cone. The diameter of the base of the cone and cylinder is 2 feet.

What is the volume of the stone that the sculptor must remove? Round your answer to the nearest hundredth.
10. STAGES A stage has the form of a square pyramid with the top sliced off along a plane parallel to the base. The side length of the top square is 12 feet and the side length of the bottom square is 16 feet. The height of the stage is 3 feet.

a. What is the volume of the entire square pyramid that the stage is part of?

b. What is the volume of the top of the pyramid that is removed to get the stage?

c. What is the volume of the stage?
12.6 Warm Up

1. Find the volume of the pyramid. Round to the nearest tenth.

2. Find the volume of the cone. Round to the nearest tenth.

Common Core State Standards
G.GMD.1 Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone.
G.GMD.3 Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.

Mathematical Practices
1. Make sense of problems and persevere in solving them.
6. Attend to precision.

Learning Targets
• Students will be able to find surface areas of spheres.
• Students will be able to find volumes of spheres.

Section 12.6 Notes: Surface Areas and Volumes of Spheres

KeyConcept Surface Area of a Sphere

<table>
<thead>
<tr>
<th>Words</th>
<th>The surface area S of a sphere is $S = 4\pi r^2$, where r is the radius.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Symbols</td>
<td></td>
</tr>
</tbody>
</table>

Example 1: Find the surface area of the sphere. Round to the nearest tenth and label the units.

A plane can intersect a sphere in a point or in a circle. If the circle contains the center of the sphere, the intersection is called a ______________________. The endpoints of a diameter of a great circle are called the _________.

Since a great circle has the same center as the sphere and its radii are also radii of the sphere, it is the largest circle that can be drawn on a sphere. A great circle separates a sphere into two congruent halves, called ________________.
**Example 2:** Find the surface area of the hemisphere. Label the units.

**Example 3:** Find the surface area of the sphere if the circumference of the great circle is $10\pi$.

**Example 4:** Find the surface area of the sphere if the area of the great circle is approximately 160 square meters.

**Example 5:** Find the volume of the sphere. Round to the nearest hundredth and label the units.

**Example 6:** Find the volume of the sphere with a great circle that has a circumference of $30\pi$ centimeters. Round to the nearest tenth.

**Example 7:** Find the volume of the hemisphere with a diameter of 6 ft.

**Example 8:** RECESS The jungle gym outside of Jada’s school is a perfect hemisphere. It has a volume of $4,000\pi$ cubic feet. What is the diameter of the jungle gym?
**New Vocabulary** Label the parts of the sphere pictured below.

**Challenge Problems**

A cube has a volume of 216 cubic inches. Find the volume of a sphere that is circumscribed about the cube. Round to the nearest tenth.

Determine whether the following statement is true or false. If true, explain your reasoning. If false, provide a counterexample.

*If a sphere has radius r, there exists a cone with radius r having the same volume.*

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**Learning Target Checklist**

- Students will be able to find surface areas of spheres.
- Students will be able to find volumes of spheres
Geometry
12.6 Worksheet

Find the surface area of each sphere or hemisphere. Round to the nearest tenth.
1. SA=______________  
   ![Image of a sphere with a radius of 6.5 cm]

2. SA=______________
   ![Image of a sphere with a radius of 89 ft]

3. hemisphere: radius of great circle = 8.4 in.

4. sphere: area of great circle \( \approx 29.8 \text{ m}^2 \)

Find the volume of each sphere or hemisphere. Round to the nearest tenth.
5. V=______________  
   ![Image of a sphere with a circumference of 12.32 ft]

6. V=______________
   ![Image of a hemisphere with a diameter of 32 m]

7. hemisphere: diameter = 18 mm

8. sphere: circumference \( \approx 36 \text{ yds} \)

9. sphere: radius = 12.4 in.

10. ORANGES Mandy cuts a spherical orange in half along a great circle. If the radius of the orange is 2 inches, what is the area of the cross section that Mandy cut? Round your answer to the nearest hundredth.

11. BILLIARDS A billiard ball set consists of 16 spheres, each \( 2 \frac{1}{4} \) inches in diameter. What is the total volume of a complete set of billiard balls? Round your answer to the nearest thousandth of a cubic inch.
12. **MOONS OF SATURN** The planet Saturn has several moons. These can be modeled accurately by spheres. Saturn’s largest moon Titan has a radius of about 2575 kilometers. What is the approximate surface area of Titan? Round your answer to the nearest tenth.

13. **THE ATMOSPHERE** About 99% of Earth’s atmosphere is contained in a 31-kilometer thick layer that enwraps the planet. The Earth itself is almost a sphere with radius 6378 kilometers. What is the ratio of the volume of the atmosphere to the volume of Earth? Round your answer to the nearest thousandth.

14. **CUBES** Marcus builds a sphere inside of a cube. The sphere fits snugly inside the cube so that the sphere touches the cube at one point on each side. The side length of the cube is 2 inches.

   a. What is the surface area of the cube?

   b. What is the surface area of the sphere? Round your answers to the nearest hundredth.

   c. What is the ratio of the surface area of the cube to the surface area of the sphere? Round your answer to the nearest hundredth.