**7-7 Study Guide and Intervention**

**Base e and Natural Logarithms**

The irrational number $e \approx 2.71828\ldots$ often occurs as the base for exponential and logarithmic functions that describe real-world phenomena.

<table>
<thead>
<tr>
<th>Natural Base e</th>
<th>As $n$ increases, $(1 + \frac{1}{n})^n$ approaches $e \approx 2.71828\ldots$.</th>
</tr>
</thead>
</table>

The functions $f(x) = e^x$ and $f(x) = \ln x$ are inverse functions.

<table>
<thead>
<tr>
<th>Inverse Property of Base e and Natural Logarithms</th>
<th>$e^{\ln x} = x$ \hspace{1cm} $\ln e^x = x$</th>
</tr>
</thead>
</table>

Natural base expressions can be evaluated using the $e^x$ and $\ln$ keys on your calculator.

**Example 1** Write a logarithmic equation equivalent to $e^{2x} = 7$.

$e^{2x} = 7 \rightarrow \log_e 7 = 2x$

$2x = \ln 7$

**Example 2** Write each logarithmic equation in exponential form.

a. $\ln x \approx 0.3345$

$\ln x \approx 0.3345 \rightarrow \log_e x \approx 0.3345$

$x \approx e^{0.3345}$

b. $\ln 42 = x$

$\ln 42 = x \rightarrow \log_e 42 = x$

$42 = e^x$

**Exercises**

Write an equivalent exponential or logarithmic equation.

1. $e^{15} = x$

$\ln x = 15$

2. $e^{3x} = 45$

$3x = \ln 45$

3. $\ln 20 = x$

$e^x = 20$

4. $\ln x = 8$

$x = e^8$

5. $e^{-5x} = 0.2$

$-5x = \ln 0.2$

6. $\ln (4x) = 9.6$

$4x = e^{9.6}$

7. $e^{8.2} = 10x$

$10x = 8.2$

8. $\ln 0.0002 = x$

$e^x = 0.0002$

Evaluate each logarithm to the nearest ten-thousandth.

9. $\ln 12,492$

$9.4328$

10. $\ln 50.69$

$3.9257$

11. $\ln 9275$

$9.1351$

12. $\ln 0.835$

$-0.1803$

13. $\ln 943 - \ln 181$

$1.6506$

14. $\ln 67 + \ln 103$

$8.8394$

15. $\ln 931 \cdot \ln 32$

$23.6927$

16. $\ln (139 - 45)$

$4.5433$
Equations and Inequalities with e and ln All properties of logarithms from earlier lessons can be used to solve equations and inequalities with natural logarithms.

Example
Solve each equation or inequality.

a. \(3e^{2x} + 2 = 10\)

\[
3e^{2x} + 2 = 10 \quad \text{Original equation}
\]

\[
3e^{2x} = 8 \quad \text{Subtract 2 from each side.}
\]

\[
e^{2x} = \frac{8}{3} \quad \text{Divide each side by 3.}
\]

\[
\ln e^{2x} = \ln \frac{8}{3} \quad \text{Property of Equality for Logarithms}
\]

\[
2x = \ln \frac{8}{3} \quad \text{Inverse Property of Exponents and Logarithms}
\]

\[
x = \frac{1}{2} \ln \frac{8}{3} \quad \text{Multiply each side by \(\frac{1}{2}\).}
\]

\[
x \approx 0.4904 \quad \text{Use a calculator.}
\]

b. \(\ln (4x - 1) < 2\)

\[
\ln (4x - 1) < 2 \quad \text{Original inequality}
\]

\[
e^{\ln (4x - 1)} < e^2 \quad \text{Write each side using exponents and base e.}
\]

\[
0 < 4x - 1 < e^2 \quad \text{Inverse Property of Exponents and Logarithms}
\]

\[
1 < 4x < e^2 + 1 \quad \text{Addition Property of Inequalities}
\]

\[
\frac{1}{4} < x < \frac{1}{4}(e^2 + 1) \quad \text{Multiplication Property of Inequalities}
\]

\[
0.25 < x < 2.0973 \quad \text{Use a calculator.}
\]

Exercises
Solve each equation or inequality. Round to the nearest ten-thousandth.

1. \(e^{4x} = 120\) \[1.1969\]

2. \(e^x \leq 25\) \[x \leq 3.2189\]

3. \(e^{x-2} + 4 = 21\) \[4.8332\]

4. \(\ln 6x \geq 4\) \[x \geq 9.0997\]

5. \(\ln (x + 3) - 5 = -2\) \[17.0855\]

6. \(e^{-ax} \leq 50\) \[x \geq -0.4890\]

7. \(e^{4x-1} - 3 = 12\) \[0.9270\]

8. \(\ln (5x + 3) = 3.6\) \[6.7196\]

9. \(2e^{3x} + 5 = 2\) \[\text{no solution}\]

10. \(6 + 3e^{x+1} = 21\) \[0.6094\]

11. \(\ln (2x - 5) = 8\) \[1492.9790\]

12. \(\ln 5x + \ln 3x > 9\) \[x > 23.2423\]